I $f(x) = \frac{2x-1}{x+2}$ $g(x) = \frac{2x-1}{x-3}$

A) $B(x) = \frac{1}{\frac{1}{f}+\frac{1}{g}}$ $\frac{x+2}{2x-1} + \frac{x-3}{2x-1}$ $\frac{2x-1}{2x-1} \geq 1$

$x \neq \{-2, 3, \frac{1}{2}\}$ $x \in \mathbb{R} - 3$ use the rules.

B) $B(x) = \sqrt{-\frac{f}{g}}$ $\frac{f}{g} = \frac{\frac{1}{x+2}}{\frac{1}{x-3}} = \frac{x-3}{x+2}$

No $\lim_{x \to 3}$

$- \circ + \circ -$ on $-2, 0, 3$

2 pts

$-2 < x < 3$; $x \neq \frac{1}{2}$

II A) $\left|\frac{x+2}{x-3}\right| \geq 1$

I II III B on $-2, 0, 3$

I, II $\frac{x+2}{x-3} \geq 1$

$x+2 \leq x-3$

$2 \leq -3$

Reject not true

II $\frac{x+2}{-(x-3)} \geq 1$

$x+2 \geq -x+3$

$2x \geq 3$

$x \geq \frac{1}{2}$

has "collapse"

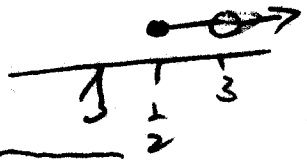
III $\frac{x+2}{x-3} \geq 1$

$x+2 \geq x-3$

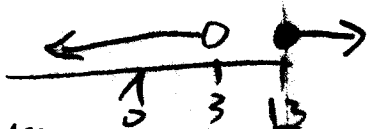
$2 \geq 3$

True

$-1 x \geq \frac{1}{2}$, $x \neq 3$



B) $\frac{1}{2x-3} \leq 4$



I $1 \geq 8x - 12$ no intersect II

"extra" negation -2

$13 \geq 8x$

$\frac{13}{8} \geq x$ $\therefore x < \frac{3}{2}$

II $1 \leq 8x - 12$

$\frac{13}{8} \leq x$

$x < \frac{3}{2}$ or $x \geq \frac{13}{8}$

III A) $\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = f'(a)$

$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$

B) $f(x) = |x+2|$

$\lim_{x \to -2} \frac{|x+2|-0}{x+2}$

$\lim_{x \to -2^-} \frac{-(x+2)}{x+2}$ $\lim_{x \to -2^+} \frac{x+2}{x+2}$

$-1 \neq 1$

$\therefore$ limit DNE

Since limit is DNE of Def of Der, Der DNE.

IV $f(x) = 2x^2 + 3$

$\lim_{h \to 0} \frac{2(x+h)^2+3-(2x^2+3)}{h}$

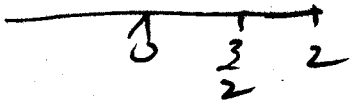
$\lim_{h \to 0} \frac{1}{h}\left[2(x^2+2xh+h^2)+3-2x^2-3\right]$

$\lim_{h \to 0} \frac{1}{h}\left[2x^2+4xh+2h^2-2x^2\right]$

$\lim_{h \to 0} 4x+2h$

$4x = f'(x)$

VII $f(x) = |2x-3| - |x-2|$



I $-(2x-3)+x-2$

$-2x+3+x-2$

$-x+1$

II $2x-3+x-2$

$3x-5$

III $2x-3-x+2$

$x-1$

$$f(x) = \begin{cases} -x+1 & \text{if } x < \frac{3}{2} \\ 3x-5 & \text{if } \frac{3}{2} \le x < 2 \\ x-1 & \text{if } x \ge 2 \end{cases}$$

$$f'(x) = \begin{cases} -1 & \text{if } x < \frac{3}{2} \\ 3 & \text{if } \frac{3}{2} < x < 2 \\ 1 & \text{if } x > 2 \end{cases}$$

Note f' DNE for $x = \frac{3}{2}$ and $x = 2$.

V

4) $\lim_{x\to\infty} \frac{\sqrt{3x^2-4}}{x+2}$

$\lim_{x\to\infty} \frac{x\sqrt{3-\frac{4}{x^2}}}{x(1+\frac{2}{x})}$

$2 \div 3 \to -\sqrt{3}$

B) $\lim_{x\to\infty} \frac{-2x+4}{9x^2+6} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{-\frac{2}{x}+\frac{4}{x^2}}{9+\frac{6}{x^2}}$

$\frac{0}{9} = 0$

C) $\lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2+4} \left(\frac{\sqrt{x^2+x}+\sqrt{x^2+4}}{\sqrt{x^2+x}+\sqrt{x^2+4}}\right)$

$\lim_{x\to\infty} \frac{x^2+x-x^2-4}{\sqrt{x^2+x}+\sqrt{x^2+4}}$

$\lim_{x\to\infty} \frac{x(1+\frac{4}{x})}{x(\sqrt{1+\frac{1}{x}}+\sqrt{1+\frac{4}{x}})}$

$\lim_{x\to\infty} \frac{1}{1+1} = \frac{1}{2}$

D) $\lim_{x\to -3} \frac{x^3+27}{9-x^2}$

$\lim_{x\to -3} \frac{(3+x)(9+3x+x^2)}{(3+x)(3-x)}$

$\frac{9+9+9}{6} = \frac{9}{2}$

E) $\lim_{x\to 9} \frac{6+2\sqrt{x}}{x-9}$

$\lim_{x\to 9} \frac{2(3+\sqrt{x})}{(\sqrt{x}+3)(\sqrt{x}-3)}$

$\frac{-2}{3+3} = \frac{-1}{3}$

F) $\lim_{x\to 0^+} x\sqrt{\frac{4}{x^2}-1}$

$\lim_{x\to 0^+} \frac{x}{x}\sqrt{4-x^2}$

$1 \cdot \sqrt{4} = 2$

G) $\lim_{x\to 0} \frac{|x|-x}{x}$

$\lim_{x\to 0^-} \frac{-2x}{x} = -1$

$\lim_{x\to 0^+} \frac{0}{x} = 0$ DNE ($-1 \neq 2$)

H) $\lim_{x\to -3} |-2| = 2$

I) $\lim_{x\to 8} \frac{\sqrt{x}}{4} = \frac{\sqrt{8}}{4} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$

Extra $p > \sqrt{a}$ $Q = \frac{p+\frac{a}{p}}{2}$

$p > \frac{a}{\sqrt{a}}$ Def of $\sqrt{a}$

$p > \frac{a}{\sqrt{a}} > \frac{a}{p}$ $\div$ Larger #

$2p > p + \frac{a}{\sqrt{a}} > p + \frac{a}{p}$ add p

$\rightsquigarrow$ $p > \frac{p+\frac{a}{p}}{2} \div 2$

$p > Q$ $Q \in D$.

1995

100	0
90	1
80	7
70	5
60	3
59	2

90 → 47

$\frac{1338}{18} = 74.3$

1 Non Honors Student (got the 47 of course)

many very poor on limits. Get out analysis limits Dittos and handed out as HW!

more Part D of absolute Bliss to After limit work.

No changes in EXAM. It's great!

9/20/2000

100	0
90	3
80	12
70	10
60	0
59	3

98 → 52

$\frac{2273}{28} = 81.2$

84.3 w/o the bottom 3.

This was the year I had the hexagon fixed. They did pretty good!

Advanced Placement Calculus

Name Col #1

Copy original problem.

Per _____ Date 9/27/95

Convince *me* that you understand the concept!

No decimals.

First Exam

2000

I Given $f(x) = \frac{2x-1}{x+2}$, $g(x) = \frac{2x-1}{x-3}$ (5 pts ea)

a) $B(x) = \frac{1}{\frac{1}{f(x)} + \frac{1}{g(x)}}$. Find D_B b) $C(x) = \sqrt{-\frac{f(x)}{g(x)}}$. Find D_C

II Solve for all $x \in \Re$. Graph solution on a number line. (10 pts ea)

a) $\frac{|x+2|}{|x-3|} \geq 1$ b) $\frac{1}{2x-3} \leq 4$

III The "definition of the derivative" is generally written in one of two forms. Both forms require the use of limits. (15 pts total)

a) State **BOTH** of these "definitions."

b) Using one of your responses in part A, PROVE that $f'(-2)$ does not exist when $f(x) = |x+2|$

c) Using the other of your responses in part A, find the derivative of $f(x) = 2x^2 + 3$

IV Given: $f(x) = |2x-3| - |x-2|$ (10 pts total)

a) Write $f(x)$ as a piece-meal function. b) Determine $f'(x)$.

V Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not. (5 pts ea)

a) $\lim_{x\to-\infty} \frac{\sqrt{3x^2-4}}{x+2}$ b) $\lim_{x\to\infty} \frac{-2x+4}{9x^2+6}$ c) $\lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2+4}$

d) $\lim_{x\to-3} \frac{x^3+27}{9-x^2}$ e) $\lim_{x\to9} \frac{6-2\sqrt{x}}{x-9}$ f) $\lim_{x\to0^+} x\sqrt{\frac{4}{x^2}-1}$

g) $\lim_{x\to0} \frac{|x|-x}{x}$ h) $\lim_{x\to-3} |-2|$ i) $\lim_{x\to8^-} \frac{\sqrt{x}}{4}$

Extra Credit ---------- 5 pts ----------

Given $P > \sqrt{a}$ and $Q = \frac{P + \frac{a}{P}}{2}$. Prove: $P > Q$

Advanced Placement Calculus

Name _____

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept!

95

First Exam

I What is the Domain ONLY of $f(x)=\dfrac{1}{\sqrt{6x^2+11x+3}}$ (5 pts)

II Given: $\lim\limits_{x\to a} f(x)=L$ (10 pts total)

a) Translate the "given" calculus-ese into English.

b) Fully explain the meaning and implications of the "given."

III Solve for all $x \in \Re$. Graph solution on a number line. $\dfrac{|x+2|}{|x-3|}\geq 1$ (10 pts)

IV The "definition of the derivative" is generally written in one of two forms. Both forms require the use of limits. (20 pts total)

a) State **BOTH** of these "definitions."

b) Using one of your responses in part A, PROVE that $f'(-2)$ does not exist when $f(x)=|x-2|$.

c) Using the other of your responses in part A, find the derivative of $f(x)=2x^2+3$

V Given: $f(x)=|2x-3|-|x-2|$ (10 pts total)

a) Write $f(x)$ as a piece-meal function. b) Determine $f'(x)$.

VI Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not. (5 pts ea)

a) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{3x^2-4}}{x+2}$ b) $\lim\limits_{x\to\infty}\dfrac{-2x+4}{9x^2+6}$ c) $\lim\limits_{x\to\infty}\sqrt{x^2+x}-\sqrt{x^2+4}$

d) $\lim\limits_{x\to-3}\dfrac{x^3+27}{9-x^2}$ e) $\lim\limits_{x\to 9}\dfrac{6-2\sqrt{x}}{x-9}$ f) $\lim\limits_{x\to 0^+} x\sqrt{\dfrac{4}{x^2}-1}$

g) $\lim\limits_{x\to 0}\dfrac{|x|-x}{x}$ h) $\lim\limits_{x\to -3}|-2|$ i) $\lim\limits_{x\to 8^-}\dfrac{\sqrt{x}}{4}$

Extra Credit ----------5 pts ----------

Given $P>\sqrt{a}$ and $Q=\dfrac{P+\frac{a}{P}}{2}$. Prove: $P > Q$

a) $f(x) = |x + |x+1||$

I 0 II

-1 $-\frac{1}{2}$ 0 1

I $|x - (x-1)|$

1

clear too much -3

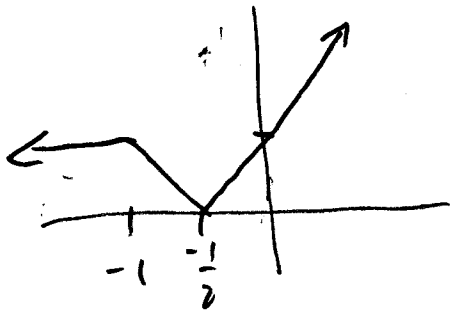
II $|x + x + 1|$

$|2x+1|$

$-(2x+1)$

$2x+1$

$$f(x) = \begin{cases} 1 & \text{if } x < -1 \\ -2x-1 & \text{if } -1 \le x < -\frac{1}{2} \\ 2x+1 & \text{if } x \ge -\frac{1}{2} \end{cases}$$



II, B

$|x^2| < |-5|$

$x^2 < 5$

$(x-\sqrt{5})(x+\sqrt{5}) = 0$

+ 0 − 0 +

$-\sqrt{5}$ 0 $\sqrt{5}$

$-\sqrt{5} < x < \sqrt{5}$

-2

$\sqrt{5}$ over inequality

b) $g(x) = |x| + |x-1|$

I 0 II 0 III

I $-x-(x-1)$

$-2x+1$

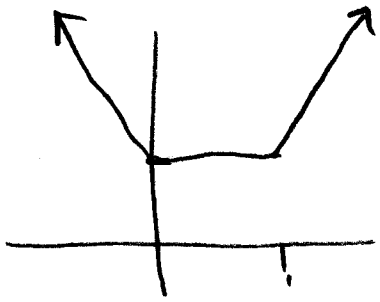
II $x - x + 1$

1

III $x + x - 1$

$2x-1$

$$g(x) = \begin{cases} -2x+1 & x < 0 \\ 1 & 0 \le x < 1 \\ 2x-1 & x \ge 1 \end{cases}$$



-4

II A) $\frac{|9-2x|}{x-5} \le \frac{7}{x-5}$ ← like $|x-5|$

I

$x < \frac{9}{2}$ inside 7a

$|9-2x| \ge 7$

$(9-2x) \ge 7$

$2 \ge 2x$

$1 \ge x$

II

$\frac{9}{2} \le x < 5$

$-(9-2x) \ge 7$

$2x \ge 16$

$x \ge 8$

Reject out of interval

$x > 5$

$-(9-2x) \le 7$

$2x \le 16$

$x \le 8$

III $f(x) = \frac{x^2+2x}{x^2+x-2} = \frac{x(x+2)}{(x+2)(x-1)} = \frac{x}{x-1}$

A) $f(f(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1}$

$\frac{x}{x-1} \overset{\text{set}}{=} -2$

$x = -2x + 2$

$3x = 2$

So $x = \frac{2}{3}$ = true

$\therefore x \neq \{1, -2, \frac{2}{3}\}$

omit

-3

$= \frac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}}$

$= \frac{x}{x-x+1}$

$= x$

2 of 3 x-1 set

identity.

B) $\lim_{h \to 0} \left(\frac{x+h}{x+h-1} - \frac{x}{x-1}\right)\frac{1}{h}$

$\lim_{h \to 0} \frac{1}{h}\left(\frac{x^2 - h - x + xh - x^2 + x}{(x-1)(x+h-1)}\right)$

$\lim_{h \to 0} \frac{1}{h} \frac{-1-1-h}{(x-1)(x+h-1)}$

$\lim_{h \to 0} \frac{-1}{(x-1)(x+h-1)}$

$\frac{-1}{(x-1)^2}$

IV

a) $\lim_{x\to-\infty} \frac{\sqrt{x^2-4}}{3x+4}$ 2 pts

$\lim_{x\to-\infty} \frac{-x\sqrt{1-\frac{4}{x^2}}}{x(3+\frac{4}{x})}$

$-\frac{1}{3}$

b) $\lim_{x\to\infty} \left(\frac{x^2-\sqrt{x}}{x^2+x\sqrt{x}}\right)^{\frac{1}{x^2}}$

$\lim_{x\to\infty} \left(\frac{1-\frac{1}{x^{\frac{3}{2}}}}{1+\frac{1}{x^{\frac{3}{2}}}}\right)$

$15x^2+x-3$

$(5x\ 1)$

c) $\lim_{x\to\infty} \sqrt{x^2+x}-\sqrt{x^2+4}$ $\frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}$

$\lim_{x\to\infty} \frac{x^2+x-x^2-4}{\sqrt{x^2+x}+\sqrt{x^2+4}}$

$\lim_{x\to\infty} \frac{x-4}{x\left(\sqrt{1+\frac{1}{x}}+\sqrt{1+\frac{4}{x}}\right)}$

$\lim_{x\to\infty} \frac{x(1-\frac{4}{x})}{x(\sqrt{1+\frac{1}{x}}+\sqrt{1+\frac{4}{x}})}$

$\frac{1}{\sqrt{1}+1}=\frac{1}{2}$

d) $\lim_{x\to0} \left[\frac{3+2x}{x+5x^2}-\frac{5}{x}\right]$

$\lim_{x\to0} \left[\frac{3+2x-3+15x}{x(1+5x)}\right]$

$\lim_{x\to0} \frac{-13x}{x(1+5x)}=-13$

e) $\lim_{x\to-2} \frac{\sqrt{x^2-2}}{x+2}$

$\lim_{x\to-2} \frac{|x|-2}{x+2}$

$\lim_{x\to-2} \frac{-x-2}{x+2}$

$\lim_{x\to-2} \frac{-(x+2)}{x+2}=-1$

f) $\lim_{x\to4^+} \frac{1}{x}-\frac{1}{4}$

$\frac{1}{4}-\frac{1}{4}=0$

g) $\lim_{x\to4^+} \left[\left(\frac{1}{x}-\frac{1}{4}\right)\frac{1}{x-4}\right]$

$\lim_{x\to4} \left[\frac{4-x}{4x}\cdot\frac{1}{x-4}\right]$

$\lim_{x\to4} -\frac{1}{4x}=\frac{-1}{16}$

h) $\lim_{x\to4^+} \left[\left(\frac{1}{x}-\frac{1}{4}\right)(x-2)\right]$

$0\,(2)$

0

i) $\lim_{x\to4^+} \left[\left(\frac{1}{x}-\frac{1}{4}\right)\left(\frac{1}{x-4}\right)^2\right]$

$\lim_{x\to4} \left[\frac{4-x}{4x}\cdot\frac{1}{x-4}\cdot\frac{1}{x-4}\right]$

$\lim_{x\to4^+} \frac{-1}{4x(x-4)}$

$\to-\infty$

No Limit.

EXTRA. Jake 1 Lap = 40 sec

$\frac{1}{40}$ Lap = 1 sec.

$\frac{3}{1}\cdot\frac{1}{40}=\frac{3}{8}$

Jake means $\frac{3}{8}$ of a Lap and Ted does $\frac{5}{8}$ of a Lap

$\frac{8}{5}\cdot\frac{5}{8}$ Lap $=15\frac{3}{8}$ sec $\cdot\frac{8}{8}$

1 Lap $=24$ sec.

9/25/96

100	0
90	4
80	3
70	8
60	4
59	4

$\frac{1663}{23} = 72.3$

without the 8

74.2 of course... he didn't take my trig class.

Excellent exam — adding the "extra" part as indicated on hw asked will help. we do Need to make them more accountable.
they went to cruise....

I required complete sentence answers to extra...

9/20/2001

100	0			95
90	2			
80	9			
70	5			
60	5			
59	4			35

1834 73.4
25

Still excellent exam!!

Advanced Placement Calculus

Name Calc 1ST EXAM 1ST Q

Copy original problem. Per _____ Date 9/25/96

Convince *me* that you understand the concept! 9/20/2001

No decimals.

First Exam

I Write as a piece-wise function then sketch. (10 pts ea)

a) $f(x) = \left|x + \left|x + 1\right|\right|$ b) $g(x) = |x| + |x - 1|$

II Solve for all $x \in \Re$. Graph solution on a number line. (10 pts ea)

a) $\dfrac{|9 - 2x|}{x - 5} \leq \dfrac{7}{x - 5}$ b) $|x^2| < |-5|$

III Given: $f(x) = \dfrac{x^2 + 2x}{x^2 + x - 2}$ (15 pts tot)

a) Solve $\forall \; x \in \Re : \; f(f(x)) = x$

b) Using the definition of the derivative ($h \to 0$), determine $f'(x)$.

IV Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not. (5 pts ea)

a) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 - 4}}{3x + 4}$ b) $\lim\limits_{x \to \infty} \left(\dfrac{x^2 - \sqrt{x}}{x^2 + x\sqrt{x}}\right)$ c) $\lim\limits_{x \to \infty} \sqrt{x^2 + x} - \sqrt{x^2 + 4}$

d) $\lim\limits_{x \to 0} \left[\dfrac{3 + 2x}{x + 5x^2} - \dfrac{3}{x}\right]$ e) $\lim\limits_{x \to -2} \dfrac{\sqrt{x^2} - 2}{x + 2}$ f) $\lim\limits_{x \to 4^+} \left(\dfrac{1}{x} - \dfrac{1}{4}\right)$

g) $\lim\limits_{x \to 4^+} \left[\left(\dfrac{1}{x} - \dfrac{1}{4}\right)\left(\dfrac{1}{x - 4}\right)\right]$ h) $\lim\limits_{x \to 4^+} \left[\left(\dfrac{1}{x} - \dfrac{1}{4}\right)(x - 2)\right]$ i) $\lim\limits_{x \to 4^+} \left[\left(\dfrac{1}{x} - \dfrac{1}{4}\right)\left(\dfrac{1}{x - 4}\right)^2\right]$

Extra Credit ---------- 5 pts ----------

Jake can run around a circular track in forty seconds. Jed, running in the opposite direction, meets Jake [illegible]. How long does it take Jed to run around the entire track, expressed in seconds?

I $f(x)=\sqrt{\dfrac{x^2-x-2}{x^2+x-6}}$

factored wrong $=\sqrt{\dfrac{(x-2)(x+1)}{(x-2)(x+3)}}$

+ 0 − 0 + ⊗ +

−3 −1 0 2

D_f $x<-3$ or $x\geq -1$, $x\neq 2$ 2 pts

III a) $\left|\dfrac{x-2}{x+2}\right|>1$

I $x-2<x+2$

$-2<2$

OR

II $-(x-2)>x+2$

$-x+2>x+2$

$0>2x$

$0>x$

IV $x-2>x+2$

$-2>2$

Reject

$\therefore x<0,\ x\neq -2$

V $f(x)=|2x+3|-|4-x|$

$a)\ f(x)=\begin{cases} -x-7 & \text{if } x<-\frac{3}{2} \\ 3x-1 & \text{if } -\frac{3}{2}\leq x<4 \\ x+7 & \text{if } x\geq 4 \end{cases}$

I $-2x-3-4+x$

$-x-7$

II $2x+3-4+x=3x-1$

III $2x+3+4-x$

$x+7$

VI $\lim_{x\to 1}\dfrac{f(x)-0}{x-1}$

$\lim_{x\to 1^-}\dfrac{x^2-1}{x-1}$ $\lim_{x\to 1^+}\dfrac{\sqrt{x}-1}{x-1}$

$\lim_{x\to 1^-}\dfrac{(x+1)(x-1)}{x-1}$ $\lim_{x\to 1^+}\dfrac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}$ VC

2 one sided only $\frac{1}{2}$

$f'(1)$ DNE (L) $\neq$ (R)

II $f(x)=3x^4+2x^3$ $(-1,1)$

$f'(x)=12x^3+6x^2$

$6x^2(2x+1)$

$6(-1)=-6=m$

$y-1=-6(x+1)$

$y=-6x-5$

III b) $\dfrac{5}{1-x}<3$ one sided only −6

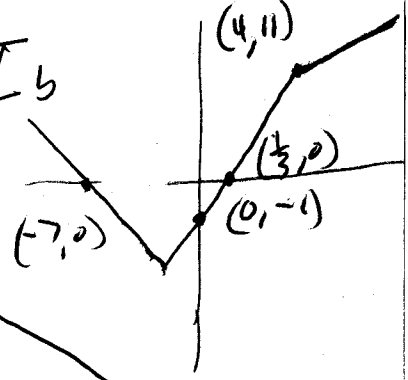
−1 $\frac{-2}{3}$ 1

if $x<-1$: $5<3-3x$, $2<-3x$, $-\frac{2}{3}>x$

if $x>1$: $5>3-3x$, $2>-3x$, $-\frac{2}{3}<x$ and $x>1$ so $x>1$

$\therefore\ x<-\frac{2}{3}$ or $x>1$

V b)



$f'(x)=\begin{cases} -1 & \text{if } x<-\frac{3}{2} \\ 3 & \text{if } -\frac{3}{2}<x<4 \\ 1 & \text{if } x>4 \end{cases}$

#11

a) $\lim_{x\to 0^-} \frac{2x^2+3x}{|x|}$

$\lim_{x\to 0^-} \frac{x(2x+3)}{-x}$

-3

b) $\lim_{x\to -3} \frac{x^3+27}{x+3}$

$\lim_{x\to -3} \frac{(x+3)(x^2-3x+9)}{(x+3)}$

No sum of cubes

$-2 \quad 9+9+9$

27

c) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$

$\lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)}$

$\frac{1}{2+2} = \frac{1}{4}$

d) $\lim_{x\to 0} \frac{x^2+\frac{1}{x}}{x^2-\frac{1}{x}}$

$\lim_{x\to 0} \frac{x^3+1}{x^3-1}$

-1

e) $\lim_{x\to 0} (1-3x)^{\frac{2}{x}}$

$\lim_{x\to 0} \left[(1+(-3x))^{-\frac{1}{3x}}\right]^{-6}$

$\frac{1}{e^6}$

f) $\lim_{x\to\infty} \sqrt{x^2+x}-\sqrt{x^2+4} \cdot \frac{\sqrt{\ }+\sqrt{\ }}{\sqrt{\ }+\sqrt{\ }}$

$\lim_{x\to\infty} \frac{x^2+x-x^2-4}{\sqrt{x^2+x}+\sqrt{x^2+4}}$

$\lim_{x\to\infty} \frac{x(1-\frac{4}{x})}{x(\sqrt{1+\frac{1}{x}}+\sqrt{1+\frac{4}{x}})}$

$\frac{1}{1+1} = \frac{1}{2}$

$f(x) = \begin{cases} 4-x & \text{if } x<1 \\ \sqrt{6} & \text{if } x=1 \\ 2+x^2 & \text{if } x>1 \end{cases}$

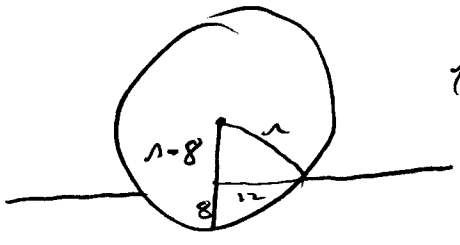
g) $\lim_{x\to 1} f(x) =$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} 2+x^2$

$\lim_{x\to 1^-} 4-x = 3 \qquad 3$

$\therefore \lim_{x\to 1} f(x) = 3$

Extra.



$r^2 = (r-8)^2 + 12^2$

$r^2 = r^2 - 16r + 64 + 144$

$16r = 208$

$r = 13$

$\therefore$ Ball radius is 13 inches.

	4	5	T
100 ↑	0	1	1
90	2	4	6
80	5	5	10
70	5	4	9
60	4	2	6
59	3	2	5

100

$\frac{1329}{18}$ $\frac{1449}{18}$

73.8 80.5 77.2

29

obviously, the 77 wasn't in my trig class!

Advanced Placement Calculus

Name Calc 1st Exam

Copy original problem.

Per _____ . Date 2002

Convince *me* that **you** understand the concept!

Quarter # 1 Exam # 1

I Given $f(x) = \sqrt{\dfrac{x^2 - x - 2}{x^2 + x - 6}}$. Determine the domain of $f(x)$. Justify. (10 pts)

II Given $f(x) = 3x^4 + 2x^3$. Determine the equation of the tangent line to $f(x)$ at $(-1, 1)$. (10 pts)

III Solve $\forall \; x \in \Re$. Graph solution set on a well labeled number line. Be especially sure that the reasons for your steps are very clear. (10 pts ea)

a) $\dfrac{|x - 2|}{|x + 2|} > 1$ b) $\dfrac{5}{1 - x} < 3$

IV Using the definition of the derivative, determine $f'(1)$ where $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ \sqrt{x} - 1 & \text{if } x \geq 1 \end{cases}$ (10 pts)

V Given $f(x) = |2x + 3| - |4 - x|$. (15 pts tot)

a) Rewrite $f(x)$ as a piece-wise function.

b) Sketch the graph of $f(x)$.

c) Determine $f'(x)$

VI Find the following limits: (5 pts ea)

a) $\lim\limits_{x \to 0^-} \dfrac{2x^2 + 3x}{|x|}$ b) $\lim\limits_{x \to -3} \dfrac{x^3 + 27}{x + 3}$ c) $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$

d) $\lim\limits_{x \to 0} \dfrac{x^2 + \frac{1}{x}}{x^2 - \frac{1}{x}}$ e) $\lim\limits_{x \to 0} (1 - 3x)^{\frac{2}{x}}$ f) $\lim\limits_{x \to \infty} \sqrt{x^2 + x} - \sqrt{x^2 + 4}$

g) $\lim\limits_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4 - x & \text{if } x < 1 \\ \sqrt{6} & \text{if } x = 1 \\ 2 + x^2 & \text{if } x > 1 \end{cases}$

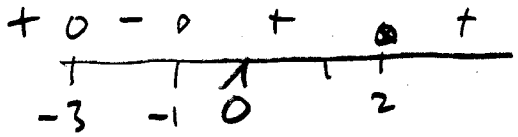
Extra Credit ---------- 5 pts ----------

A ball was floating in a lake when the lake froze. The ball was removed (without needing to break the ice), leaving a hole 24 inches across at the top and 8 inches deep at the deepest point of the indentation. What was the radius of the ball?

A P CALCULUS **1st EXAM** **2002** School Holiday 9 / 2	#1 R/C pg 1 - 9 #2 Pg 9 - 10 "Algebra Review" 1 - 15 9 / 3	#3 R/C pg 11 - 15 #4 Pg 15 - 16 1 - 15 18 - 20, 22 - 25 *Be sure your answers are reasonable!*	#5 R/C pg 21 - 25 # 6 Pg 25 - 26 1 - 12, 14 9 / 5	# 7 R/C pg 26 - 27 "Summary" # 8 Pg 27 GQ Ch 2 3 - 8 # 9 Pg 27 - 28 RE Ch 2 1 - 13 "Fenway" 9 / 6
# 10 R/C Pg 29 - 33 R/C Pg 35 - 40 #11 Pg 40 1 - 20 **You *must* use the definition of the derivative!** 9 / 9	# 12 Pg 48 14 - 19 Pg 50 GQ 1 - 7;RE 1-4,7 # 13 R/C "Summary" Pg 48-50 Image = Range pre-image = domain	# 14 Pg 50 - 52 RE 9 - 13, 15 - 17, 19 - 30 9/ 11	# 15 Pg 58 - 59 1 - 12, 16 - 18, 20 - 22,25, 29, 30 **"Window on World"**	# 16 Pg 64 1 - 5, 9 - 14
Begin "Absolute Bliss" # 17 Parts I and II $\sqrt{\star^2} = \lvert\star\rvert = \begin{cases} -\star & ;\star < 0 \\ \star & ;\star \geq 0 \end{cases}$ 9/16	Continue on "Absolute Bliss" # 18, # 19 Parts III and IV 9/17	# 20, 21 "Absolute Bliss" Part V 9/18	# 22, #23 "Absolute Bliss" Part VI 9/ 19	# 24, 25 "Absolute Bliss" Part VII **Reasons a limit might not exist:** **"Cannot Approach"** **"Oscillates"** "$\rightarrow -\infty$" "$\rightarrow +\infty$" **(L) ≠ (R)**
# 26 Pg 71 - 72 1 - 28, 30, 31 # 27 Pg 76 - 77 1 - 30	# 28 Pg 77 31 - 42 **infinite limits** 9/ 24	REVIEW Put up last year's exam. 9 / 25	**EXAM # 1** **Chapters 1 —> 4.4** 9/ 26	

I $f(x) = \sqrt{\frac{x^2-x-2}{x^2+x-6}}$

done separately. $= \sqrt{\frac{(x-2)(x+1)}{(x+3)(x-2)}}$ -1



D_f $x < -3$ or $x > -1$, $x \neq 2$) 5

II $f(x) = 3x^4 + 2x^3$ $(-1, 1)$

$f'(x) = 12x^3 + 6x^2$

$6x^2(2x+1) = 6(-1) = -6$

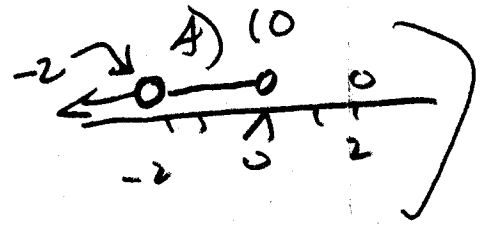
$x = -1$

$y - 1 = -6(x+1)$

$y = -6x - 5$) 5

III

A) $\frac{|x-2|}{|x+2|} > 1$



I $\frac{x-2}{x+2} > 1$

$x - 2 < x + 2$

$-2 < 2$

TRUE

II $-x+2 > x+2$

$0 > 2x$

$0 > x$

III $x - 2 > x + 2$

$-2 > 2$

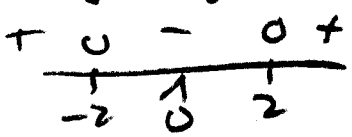
Reject.

$x < 2$

B) $|x^2| > 4$

$x^2 - 4 > 0$

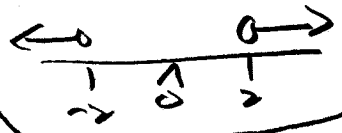
$(x-2)(x+2)$



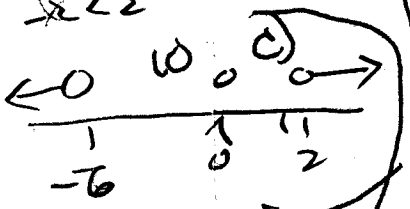
$|x| > 2$ B) 10

or

$-2 < x$ or $x > 2$



C) $|-x| + |-x| + x > 6$



I $-x - x + x > 6$

$-x > 6$

$x < -6$

II $x + x + x > 6$

$3x > 6$

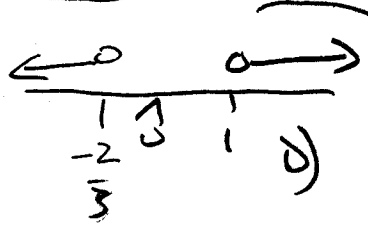
$x > 2$

D) $\frac{-5}{x-1} < 3$

I $-5 \geq 3x - 3$

$-2 \geq 3x$

$-\frac{2}{3} \geq x$



II $-5 < 3x - 3$

$-2 < 3x$

$-\frac{2}{3} < x$ But

$x > 1$

IV $f(x) = 3x^2+3$

$\lim_{h\to 0} \frac{3(x+h)^2+3-3x^2-3}{h}$

$\lim_{h\to 0} \frac{3x^2+6xh+3h^2-3x^2-3}{h}$

$\lim_{h\to 0} 6x+3h = 6x$) 5

VII 3 pts ea

1) $\lim_{x\to 3} \frac{\frac{1}{x}+\frac{1}{2}}{x-2} = \frac{\frac{1}{3}+\frac{1}{2}}{1} = \frac{5}{6}$

B) $\lim_{x\to 0} \sqrt{1-x} = 1$

c) $\lim_{x\to -3} \frac{x^3+3^3}{x+3}$

$\lim_{x\to -3} \frac{(x+3)(x^2-3x+9)}{x+3} = 27$)

V $|2x+|x-3|| < 2$

I 0 II 3 III

I $|2x-x+3| < 2$

II $|x+3| < 2$

$-x-3 < 2$

$-x < 5$

$x > -5$

III $x+3 < 2$

$x < -1$

IV $|2x+x-3| < 2$

$|3x-3| < 2$ $x > 3$

$3x-3 < 2$

$3x < 5$

$x < \frac{5}{3}$

Reject $x > 3$

5) (-5, -3, -1, 0, 3)

VI $f(x) = |2x+3| - |4-x|$

0 0

$-\frac{3}{2}$ 4

I $-(2x+3) - (4-x)$

$-2x-3-4+x$

$-x-7$

II $2x+3-4+x$

$3x-1$

III $2x+3+4-x$

$x+7$

$f(x) = \begin{cases} -x-7 & x < -\frac{3}{2} \\ 3x-1 & -\frac{3}{2} \le x < 4 \\ x+7 & 4 \le x \end{cases}$ 10 pts

D) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{}+2}{\sqrt{}+2}$

$\lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{1}{4}$)

E) $\lim_{x\to 0} \frac{x^2+\frac{1}{x}}{x^2-\frac{1}{x}}$

$\lim_{x\to 0} \frac{x^3+1}{x^3-1} = -1$)

F) $\lim_{x\to -\infty} 2^x = \frac{1}{2^{\text{large}}} = 0$)

G) $\lim_{x\to 1^+} \frac{\sqrt{x^2-1}}{\sqrt{x^3-1}}$

$\lim_{x\to 1^+} \sqrt{\frac{(x-1)(x+1)}{(x-1)(x^2+x+1)}}$

$\sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$)

H) $\lim_{x\to 1}$ ($\frac{x^3-x^2+2x-2}{x-1}$

$\lim_{x\to 1} \frac{x^2(x-1)+2(x-1)}{x-1}$

$\lim_{x\to 1} x^2+2 = 3$)

I) $\lim_{x\to 0} (1+3x)^{\frac{2}{x}}$

$\lim_{x\to 0} \left[(1+3x)^{\frac{1}{3x}}\right]^6 = e^6$)

J) $\lim_{x\to 0^-} |4-3x| = 4$)

VII $f'(x) = \begin{cases} -1 & x < -\frac{3}{2} \\ 3 & -\frac{3}{2} < x < 4 \\ 1 & x > 4 \end{cases}$ 13

$\in$ x+n

$(n-8)^2 + 12^2 = n^2$

$n^2 - 16n + 64 + 144 = n^2$

$16n = 208$

$n = 13$)

16 | 208

16

48

100	0
90	5
80	15
70	4
60	1
59	1

96

42

81.7

Excellent extra

Probably one problem too long.

I would remove IV

Put the extra 5 pts on II

Timing was excellent.

We did a nice mix of

Board work —

It helped.

Advanced Placement Caculus AB First EXAM NAME Ch 1 → 4.4

Convince ME that you know the concept ! PER _____ DATE 9/28/88

I Given : $F(x) = \sqrt{\frac{x^2-x-2}{x^2+x-6}}$ Determine the Domain of F(x). (5 pts)

II Given $F(x) = 3x^4+2x^3$. Find the equation of the tangent line at (-1, 1). (5 pts)

III Solve $\forall x \in \mathbf{R}$. Graph solution set on a number line. (10 pts ea.)

A) $\frac{|x-2|}{|x+2|} > 1$ B) $|x^2| > |(-2)^2|$ C) $|-x|+|-x|+x>6$ D) $\frac{-5}{x-1} < 3$

IV Using the definition of the derivatives where $h \to 0$, find $F'(x)$ given $F(x) = 3x^2+3$ (5 pts)

V Solve $\forall x \in \mathbf{R}$ Graph solution set on a number line. $\left|2x+|x-3|\right| < 2$ (5 pts)

VI Given : $F(x) = |2x+3|-|4-x|$ Write F(x) *and* F'(x) as different piece-meal functions. (10 pts)

VII Find the following limits : (3 pts ea)

A) $\lim_{x\to 3} \frac{\frac{1}{x}+\frac{1}{2}}{x-2}$ B) $\lim_{x\to 0} \sqrt{1-x}$ C) $\lim_{x\to -3} \frac{x^3+3^3}{x+3}$ D) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

E) $\lim_{x\to 0} \frac{x^2+\frac{1}{x}}{x^2-\frac{1}{x}}$ F) $\lim_{x\to -\infty} 2^x$ G) $\lim_{x\to 1^+} \frac{\sqrt{x^2-1}}{\sqrt{x^3-1}}$ H) $\lim_{x\to 1} \left(\frac{x^3-x^2+2x-2}{x-1} \right)$

I) $\lim_{x\to 0} (1+3x)^{\frac{2}{x}}$ J) $\lim_{x\to 0^-} |4-3x|$

EXTRA CREDIT ---- 5 PTS

A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a hole 24 inches across at the top and 8 inches deep. What was the radius of the ball?

I $f(x) = \frac{1}{\sqrt{1-x^2}}$

$1-x^2 > 0$

$x^2 < 1$

$-1 < x < 1$

II $f(x) = 27^{-\frac{1}{x}}$

$f(\frac{3}{2}) = 27^{-\frac{1}{\frac{3}{2}}}$

$= \frac{1}{27^{\frac{2}{3}}}$

$= \frac{1}{3^2} = \frac{1}{9}$

III $\lim_{x \to a} f(x) = L$

A) the limit of $f(x)$ as x approaches a is equal to some number L.

B) as x nears a from both above and below the $f(x)$ values both are approaching some numbers. those numbers are both the same and they are L.

IV $\frac{|x+3|}{|x-2|} \geq 1$

I o II o III

-3 0 2

I if $x < -3$

$\frac{-(x+3)}{-(x-2)} \geq 1$

$x+3 \leq x-2$

$3 \leq -2$

Reject.

II $-3 < x < 2$

$\frac{x+3}{-(x-2)} \geq 1$

$x+3 \geq -x+2$

$2x \geq -1$

$x \geq -\frac{1}{2}$

TRUE $-\frac{1}{2} \leq x < 2$

III $x > 2$

$\frac{x+3}{x-2} \geq 1$

$x+3 \geq x-2$

$3 \geq -2$

TRUE

$\forall x > 2$

conclusion

$-\frac{1}{2}$ 0 1 2

$x > -\frac{1}{2}, x \neq 2,$

V $f(x) = |2x+3| + |x-2|$

I o II o III

$-\frac{3}{2}$ 0 2

I if $x < -\frac{3}{2}$

$f(x) = -(2x+3) - x + 2$

$= -3x - 1$

II if $-\frac{3}{2} \leq x < 2$

$f(x) = 2x+3 - x + 2$

$= x + 5$

III if $x > 2$

$f(x) = 2x+3+x-2$

$= 3x + 1$

A)

$$f(x) = \begin{cases} -3x-1 & \text{if } x < -\frac{3}{2} \\ x+5 & \text{if } -\frac{3}{2} \leq x < 2 \\ 3x+1 & \text{if } x \geq 2 \end{cases}$$

B)

$$f'(x) = \begin{cases} -3 & \text{if } x < -\frac{3}{2} \\ 1 & \text{if } -\frac{3}{2} < x < 2 \\ 3 & \text{if } x > 2 \end{cases}$$

VI $f(x) = |x-2|$

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$\lim_{x \to 2} \frac{|x-2|}{x-2}$

$\lim_{x \to 2^+} \frac{x-2}{x-2} = 1$ $\lim_{x \to 2^-} \frac{-(x-2)}{x-2} = -1$

Since $1 \neq -1$

$\lim_{x \to 2} \frac{|x-2|}{x-2}$ DNE.

since that limit is the definition of the derivative the derivative DNE at $x = 2$.

VII

A) $\lim_{x\to -1} \frac{(x+2)}{1+x^2}$ $\quad \frac{-1+2}{1+1} = \frac{1}{2}$

B) $\lim_{x\to 2} 3 = 3$

C) $\lim_{N\to\infty} \left(1+\frac{2}{N}\right)^N$

$\lim_{N\to\infty} \left[\left(1+\frac{1}{\frac{N}{2}}\right)^{\frac{N}{2}}\right]^2 = e^2$

D) $\lim_{x\to 2} \frac{3x^2-5x-2}{x-2}$

$\lim_{x\to 2} \frac{(3x+1)(x-2)}{x-2} = 7$

E) $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} \frac{\sqrt{\ }+3}{\sqrt{\ }+3}$

$\lim_{x\to 0} \frac{x^2}{x^2\sqrt{x^2+9}+3} = \frac{1}{6}$

F) $\lim_{x\to\infty} \frac{x}{2x+1} \frac{\frac{1}{x}}{\frac{1}{x}}$

$\lim_{x\to\infty} \frac{1}{2+\frac{1}{x}} = \frac{1}{2}$

G) $\lim_{x\to 0^+} \left[\frac{3+2x}{x+5x^2} - \frac{3}{x}\right]$

$\lim_{x\to 0^+} \left[\frac{3+2x-3-15x}{x(1+5x)}\right]$

$\lim_{x\to 0^+} \left[\frac{-13}{1+5x}\right] = -13$

H) $\lim_{x\to 0} (1+3x)^{\frac{2}{x}}$

$\lim_{x\to 0} \left[(1+3x)^{\frac{1}{3x}}\right]^6$

e^6

I) $\lim_{x\to 0} \frac{2x^2+3x}{|x|}$

$\lim_{x\to 0^+} \frac{(2x^2+3x)}{-x}$

$\lim_{x\to 0^+} -(2x+3) = -3$

J) $\lim_{x\to 1} f(x)$ $\quad f(x) = \begin{cases} 4-x & \text{if } x<1 \\ -2 & \text{if } x=1 \\ 2+x^2 & \text{if } x>1 \end{cases}$

$\lim_{x\to 1^+} f(x) = 3$

$\lim_{x\to 1^-} f(x) = 3$

$\therefore \lim_{x\to 1} f(x) = 3$

Extra credit.

$f(x) = \begin{cases} x^2-1 & \text{if } x<1 \\ \sqrt{x}-1 & \text{if } x\geq 1 \end{cases}$

$\lim_{h\to 0^-} \frac{(x+h)^2+1-x^2+1}{h}$

$\lim_{h\to 0^-} \frac{x^2+2xh+h^2-x^2}{h}$

$\lim_{h\to 0^-} 2x+h = 2x\Big|_{x=1} = 2$

$\lim_{h\to 0^+} \frac{\sqrt{x+h}-1-\sqrt{x}+1}{h}$

$\lim_{h\to 0^+} \frac{\sqrt{x+h}-\sqrt{x}}{h} \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$\lim_{h\to 0^+} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0^+} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}\Big|_{x=1} = \frac{1}{2}$

Since the left and right hand limits are different $f'(1)$ DNE.

$\frac{2383}{31} = 76.9$ 9/24/84

Make changes indicated on graded exam. Would like to develop "shaving" plan for this pump.

100 → 102
90 → 101 − 2 3
99
97 3
92
88
83 − 2 5
82
80
79 − 3
78
77
76 11
75
73
71
70 − 2
69 − 2
67 − 2 4
64
69
58 5
55 − 2
44

9/27/89

10¢	1	
97		.85, 1
95		
94-2	7	as before
93		
92		Buttons 3.
90		weren't in
89		my trig
85		
84	5	2 errors
82		~~on~~ marked
81		on
76	2	opposite
71		side.
65	1	
57	1	

ADVANCED PLACEMENT CALCULUS NAME CALC Exam #1

Convince ME that YOU know the concept ! PER DATE 9/27/1989

Copy original problem. SHOW ALL WORK !

EXAM #21

I Determine the Domain of $F(x) = \dfrac{1}{\sqrt{1-x^2}}$ (5 pts)

II Given: $F(x) = (27)^{-\frac{1}{x}}$; Find $F\left(\frac{3}{2}\right)$ (5 pts)

III Given : $\lim_{x \to a} F(x) = L$ (10 pts)

A) Translate the "Given" calculus-ese into English.

B) Fully explain the meaning and significance of the "Given."

IV Solve $\forall x \in R$ and graph solution on a number line : $\dfrac{|x+3|}{|x-2|} \geq 1$ (10 pts)

V Given $F(x) = |2x+3| + |x-2|$ (10 pts)

A) Write F(x) as a "piece-meal" function.

B) Determine $F'(x)$

VI Using the definition of the derivative, (10 pts)

PROVE $F'(2)$ does not exist if $F(x) = |x-2|$

VII Evaluate the following limits, if they exist. Be sure your method is clear to me ! If a limit does not exist, explain how you know that fact. (5 pts ea)

A) $\lim_{x \to -1} \dfrac{x+2}{1+x^2}$

B) $\lim_{x \to -2^-} 3$

C) $\lim_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^x$

D) $\lim_{x \to 2} \dfrac{3x^2 - 5x - 2}{x-2}$

E) $\lim_{x \to 0} \dfrac{\sqrt{x^2+9} - 3}{x^2}$

F) $\lim_{x \to \infty} \dfrac{x}{2x+1}$

G) $\lim_{x \to 0^+} \left[\dfrac{3+2x}{x+5x^2} - \dfrac{3}{x}\right]$

H) $\lim_{x \to 0} (1+3x)^{\frac{2}{x}}$

I) $\lim_{x \to 0^-} \dfrac{2x^2 + 3x}{|x|}$

J) $\lim_{x \to 1} F(x)$ where $F(x) = \begin{cases} 4-x; \text{ if } x < 1 \\ -2; \text{ if } x = 1 \\ 2+x^2; \text{ if } x > 1 \end{cases}$

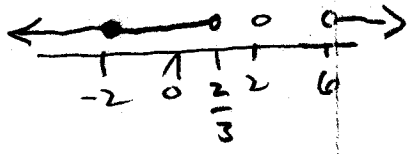
EXTRA CREDIT

Determine $F'(1)$ where $F(x) = \begin{cases} x^2 - 1; \text{ if } x < 1 \\ \sqrt{x} - 1; \text{ if } x \geq 1 \end{cases}$ using the definition of the derivative

1985

I 10 pts

A) $\left|\frac{x+2}{x-2}\right| < 2$



I $\frac{x+2}{x-2} < 2$

$x+2 > 2x-4$

$6 > x$

II $\frac{x+2}{2-x} < 2$

$x+2 < 4-2x$

$3x < 2$

$x < \frac{2}{3}$

III $\frac{x+2}{x-2} < 2$

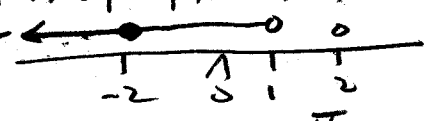
$x+2 < 2x-4$

$6 < x$

each region 2pts

B) $|x+2| - |x-2| < 2$

10 pts



I $-x-2+x-2 < 2$

$-4 < 2$

True $\forall x < -2$

II $x+2+x-2 < 2$

$2x < 2$

$x < 1$

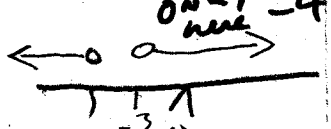
III $x+2-x+2 < 2$

$4 < 2$

Reject $\forall x \geq 2$

C) $\frac{1}{x+2} < 2$

10 pts.



only one -4

$1 > 2x+4$

$-3 > 2x$

$-\frac{3}{2} > x$

But $x < -2$

$\therefore x < -2$

$\frac{1}{x+2} < 2$

$1 < 2x+4$

$-3 < 2x$

$-\frac{3}{2} < x$

II $d = \sqrt{(b-1)^2 + (4-2)^2}$

$\sqrt{20} = \sqrt{b^2-2b+1+4}$

$20 = b^2 - 2b + 5$

$b^2 - 2b - 15 = 0$ 5 pts

$(b-5)(b+3) = 0$

$b \in \{5, -3\}$ B cannot be determined because there are 2 possible values, not conclusive

III $f(x) = 2x^2 - 3$

$\lim_{h \to 0} \frac{2(x+h)^2 - 3 - (2x^2-3)}{h}$

$\lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3 - 2x^2 + 3}{h}$ 5 pts

$\lim_{h \to 0} 4x + 2h = 4x$

IV $f(x) = \frac{x}{x-1}$

$f(f(x)) = x$

$\frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = x$

$\frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = x$ 5 pts

$x = x$

True $\forall x \neq 1$

V 5ptiea.

A) $\lim_{x\to 2} \frac{5x^3+4}{x-3} = -44$

B) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

$\lim_{x\to 2} x+2 = 4$

C) $\lim_{x\to\infty} \sqrt{\frac{3x+5}{6x-8}}$

$\lim_{x\to\infty} \sqrt{\frac{3+\frac{5}{x}}{6-\frac{8}{x}}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

D) $\lim_{x\to-\infty} \frac{4x^2-x}{2x^3-5}$

$\lim_{x\to-\infty} \frac{\frac{4}{x}-\frac{1}{x^2}}{2-\frac{5}{x^3}} = 0$

E) $\lim_{x\to-\infty} |-3| = 3$

F) $\lim_{x\to 8^-} \frac{\sqrt{x}}{4} = \frac{\sqrt{8}}{4} = \frac{\sqrt{2}}{2}$

G) $\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{3x-6}$

$\lim_{x\to\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}} = \frac{1}{3}$

$\sqrt{x^2} = x$ since $x>0$.

H) $f(x) = \begin{cases} x^2-5 & x<3 \\ \sqrt{x+13} & x>3 \end{cases}$

$\lim_{x\to 3^-} f(x) = 9-5 = 4$

$\lim_{x\to 3^+} f(x) = \sqrt{3+13} = 4$

$\therefore \lim_{x\to 3} f(x) = 4$

I) $\lim_{x\to-\infty} \frac{x-2}{x^2+2x-1}$

$\lim_{x\to-\infty} \frac{1-\frac{2}{x}}{x+2-\frac{1}{x^2}}$

$\frac{1}{-\infty} = 0$

J) $\lim_{x\to-2} \frac{x^3+8}{x+2}$

$\lim_{x\to-2} \frac{(x+2)(x^2-2x+4)}{x+2} = 12$

K) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x} \cdot \frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}$

$\lim_{x\to 0} \frac{x+3-3}{x(\sqrt{x+3}+\sqrt{3})} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

Extra.

$P > \sqrt{a}$ $\quad Q = \frac{P+\frac{a}{P}}{2}$ $\quad$ w.t.p. $Q < P$

$P > \sqrt{a} \Rightarrow \frac{a}{\sqrt{a}} > \frac{a}{P}$ $\quad$ $\frac{1}{}$ Larger Number.

$\Rightarrow$ i.e. $P > \frac{a}{P}$

$\therefore Q = \frac{P+\frac{a}{P}}{2} \Rightarrow Q < \frac{P+P}{2}$

$Q < P$

1985

100↑	1
90↑	10
80	17
70	1
60	2
54	2

85.03

101

3 of bottom 4 Scores are students from outside my trip —

good opening exam one Typo as shown in green.

with out them 88.0 average.

100	90
90	1
80	19
70	7
60	1
59	1

90 most test pts on V c, d & i

didn't use $\frac{\frac{1}{x}}{\frac{1}{x}}$ to create zeros when limit → ∞

No body got extra — no time & think.

$\frac{2276}{28} = 80.1$

Several couldn't or didn't use Lim method for $h \to 0$ III.

Still a great exam!

Put "1ST EXAM" as header

Advanced Placement Calculus AB Name Calc 1ST EXAM

Copy the original problem.

Convince *me* that **you** understand the concept. Per _____ Date 9/26/90

I Solve and graph on a number line: (10 pts ea)

A) $\left|\frac{x+2}{x-2}\right| < 2$ B) $|x+2|-|x-2| < 2$ C) $\frac{1}{x+2} < 2$

II Given the distance between (1, 2) and (**b** , 4) is $\sqrt{20}$, determine the value of **b**, if possible. If the value of **b** cannot be determined, explain the reason it cannot be determined. (5 pts)

III Given: $f(x) = 2x^2 - 3$. Using the definition of the derivative, find f ' (x). (5 pts)

IV Given: $f(x) = \frac{x}{x-1}$. Solve $\forall x \in \mathbb{R}$: $f(f(x)) = x$ (5 pts)

V Evaluate the following limits, if they exist. Be sure your method is clear to *me*. If a limit does not exist, explain *how you know that fact.* (5 pts ea)

A) $\lim_{x \to 2} \frac{5x^3+4}{x-3}$ B) $\lim_{x \to 2} \frac{x^2-4}{x-2}$ C) $\lim_{x \to \infty} \sqrt{\frac{3x+5}{6x-8}}$

D) $\lim_{x \to -\infty} \frac{4x^2-x}{2x^3-5}$ E) $\lim_{x \to -\infty} |-3|$ F) $\lim_{x \to 8^-} \frac{\sqrt{x}}{4}$

G) $\lim_{x \to \infty} \frac{\sqrt{x^2+2}}{3x-6}$ H) $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} x^2-5 & , x<3 \\ \sqrt{x+13} & , x>3 \end{cases}$

I) $\lim_{x \to -\infty} \frac{x-2}{x^2+2x-1}$ J) $\lim_{x \to -2} \frac{x^3+8}{x+2}$ K) $\lim_{x \to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$

Extra Credit 5 pts

Given: $P > \sqrt{a}$ and $Q = \frac{P + \frac{a}{P}}{2}$.

Prove: $Q < P$

I $f(x) = \dfrac{1}{\sqrt{6x^2+11x+3}}$

$(3x+1)(2x+3)$

$+ \;\; 0 \;\; - \;\; 0 \;\; +$

$-\frac{3}{2} \qquad -\frac{1}{3} \qquad 0$

5 pts

$D_f \quad x < -\frac{3}{2}$ or $x > -\frac{1}{3}$

II $\lim_{x \to a} f(x) = L$

A) translate

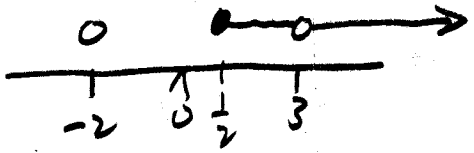
B) (L) (R) limits. $L_1 = L_2 = L$

approach L is a particular #.

"if not to plug in, then we do"

10 total

III $\dfrac{|x+2|}{|x-3|} \geq 1$



10 total

I if $x < -2$

$\dfrac{x+2}{x-3} \geq 1$

$x+2 \leq x-3$

$2 \leq -3$ reject

II if $-2 \leq x < 3$

$\dfrac{x+2}{-x+3} \geq 1$

$x+2 \geq -x+3$

$2x \geq 1$

$x \geq \frac{1}{2}$

III if $x > 3$

$\dfrac{x+2}{x-3} \geq 1$

$x+2 \geq x-3$

$2 \geq -3$

True $\forall x > 3$

IV A) $\lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ $\lim_{x \to a} \dfrac{f(x)-f(a)}{x-a}$

($f = 2x^2+3$) $\lim_{h \to 0} \dfrac{2(x+h)^2+3-2x^2-3}{h}$

total 20

$\lim_{h \to 0} \dfrac{2x^2+4xh+2h^2-2x^2}{h}$

$\lim_{h \to 0} \; 4x+2h = 4x = f'(x)$

C) $f(x) = |x-2|$

$\lim_{x \to 2} \dfrac{|x-2|-|2-2|}{x-2}$

$\lim_{x \to 2} \dfrac{|x-2|}{x-2}$

$\lim_{x \to 2^-} \dfrac{-(x-2)}{x-2} = -1$ ⟵ ≠

$\lim_{x \to 2^+} \dfrac{x-2}{x-2} = 1$ ⟵

∴ limit DNE, since limit is Def of Der, Derivative DNE.

V $f(x) = |2x+3| - |x-2|$

I o II o III

$0 \qquad -\frac{3}{2} \qquad 2$

I $-2x+3+x-2$

II $2x+3+x-2$

III $2x+3-x+2$

No derivatives at corners

$f(x) = \begin{cases} -x+1 & \text{if } x \leq \frac{3}{2} \\ 3x+5 & \frac{3}{2} < x \leq 2 \\ x-1 & \text{if } x > 2 \end{cases}$

10 total

$f'(x) = \begin{cases} -1 & \text{if } x < \frac{3}{2} \\ 3 & \frac{3}{2} < x < 2 \\ 1 & \text{if } x > 2 \end{cases}$

VI

4) $\lim_{x\to-\infty} \frac{\sqrt{3x^2-4}}{x+2}$

$\lim_{x\to-\infty} \frac{-x\sqrt{3-\frac{4}{x^2}}}{x+2}$ $\frac{\frac{1}{x}}{\frac{1}{x}}$ ✓

$\lim_{x\to-\infty} \frac{-\sqrt{3-\frac{4}{x^2}}}{1+\frac{2}{x}} = -\sqrt{3}$

c) $\lim_{x\to\infty} \sqrt{x^2+x}-\sqrt{x^2+4}$ $\frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}$

$\lim_{x\to\infty} \frac{x^2+x-x^2-4}{\sqrt{x^2+x}+\sqrt{x^2+4}}$ $\frac{\frac{1}{x}}{\frac{1}{\sqrt{x^2}}}$ ✓

$\lim_{x\to\infty} \frac{1-\frac{4}{x}}{\sqrt{1+\frac{1}{x}}+\sqrt{1+\frac{4}{x}}} = \frac{1}{2}$

f) $\lim_{x\to0} x\sqrt{\frac{4}{x^2}-1}$

$\lim_{x\to0} \sqrt{4-x^2} = 2$

✓

G) $\lim_{x\to0} \frac{|x|-x}{x}$

$\lim_{x\to0^-} \frac{-2x}{x} = -2$

$\lim_{x\to0^+} \frac{0}{x} = 0$

L ≠ R ∴ DNE ✓

13) $\lim_{x\to\infty} \frac{-2x+4}{9x^2+6}$ $\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{-\frac{2}{x}+\frac{4}{x^2}}{9+\frac{6}{x^2}} = \frac{0}{9} = 0$ ✓

D) $\lim_{x\to-3} \frac{x^3+27}{9-x^2} = \lim_{x\to-3} \frac{(x+3)x^2+3x+9}{(x+3)(3-x)}$

$\frac{9+9+9}{6} = \frac{9}{2}$ ✓

E) $\lim_{x\to9} \frac{6-2\sqrt{x}}{x-9}$ $\frac{6+2\sqrt{x}}{6+2\sqrt{x}}$

$\lim_{x\to9} \frac{36-4x}{(x-9)(6+2\sqrt{x})} = \lim_{x\to9} \frac{4(9-x)}{(x-9)(6+2\sqrt{x})}$

$\lim_{x\to9} \frac{-4}{6+2\sqrt{x}} = \frac{-4}{6+6} = -\frac{1}{3}$ ✓

H) $\lim_{x\to-3} |-2| = 2$ ✓

I) $\lim_{x\to8^-} \frac{\sqrt{x}}{4} = \frac{\sqrt{8}}{4} = \frac{\sqrt{2}}{2}$ ✓

EXTRA

$p > \sqrt{a}$ $Q = \frac{p+\frac{a}{p}}{2}$ w.t.p. $Q < p$

$p > a = \frac{a}{\sqrt{a}} > \frac{a}{p}$ ∴ Larger Number

ie $p > \frac{a}{p}$

$\therefore Q = \frac{p+\frac{a}{p}}{2} \Rightarrow Q < \frac{p+p}{2}$

$Q < p$

Q.E.D.

$p > \sqrt{a}$ Given

$p > \frac{a}{\sqrt{a}}$ Def of $\sqrt{a}$

$\frac{a}{\sqrt{a}} > \frac{a}{p}$ ∴ Larger #

$p > \frac{a}{\sqrt{a}} > \frac{a}{p}$ Since Given

Put in $2p > p+\frac{a}{p} > p+\frac{a}{p}$ add p

Larger numbers this side Larger, $p >$ $\frac{p+\frac{a}{p}}{2}$ ÷2

$p > Q$

Q.E.D.

$p > \sqrt{a}$

$p > \frac{a}{\sqrt{a}}$ Def of $\sqrt{a}$

$p > \frac{a}{\sqrt{a}} > \frac{a}{p}$ $\div$ Larger #

$2p > p + \frac{a}{\sqrt{a}} > p + \frac{a}{p}$ add p

$p > \frac{p + \frac{a}{\sqrt{a}}}{2} > \frac{p + \frac{a}{p}}{2}$ $\div 2$

$p > \quad > \frac{p + \frac{a}{p}}{2} = Q$

$p > Q$

Now WTP $Q > \sqrt{a}$

$Q = \frac{p + \frac{a}{p}}{2}$

$2Q = p + \frac{a}{p}$

$2Q < p + \frac{a}{\sqrt{a}}$

$2Q < p + \sqrt{a}$

100↑ 0

90↑ 6

80↑ 14

70↑ 27

60↑ 0

59↓ 2

93

46

$$\frac{1999}{25} = 79.96$$

Advanced Placement Calculus

Name Cal 1 ST EXAM 92

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept!

First Exam

I What is the Domain ONLY of $f(x)=\dfrac{1}{\sqrt{6x^2+11x+3}}$ (5 pts)

II Given: $\lim_{x\to a} f(x)=L$ (10 pts total)

a) Translate the "given" calculus-ese into English.

b) Fully explain the meaning and implications of the "given."

III Solve for all $x \in \Re$. Graph solution on a number line. $\dfrac{|x+2|}{|x-3|}\geq 1$ (10 pts)

IV The "definition of the derivative" is generally written in one of two forms. Both forms require the use of limits. (20 pts total)

a) State **BOTH** of these "definitions."

b) Using one of your responses in part A, PROVE that $f'(-2)$ does not exist when $f(x)=|x+2|$.

c) Using the other of your responses in part A, find the derivative of $f(x)=2x^2+3$

V Given: $f(x)=|2x-3|-|x-2|$ (10 pts total)

a) Write $f(x)$ as a piece-meal function. b) Determine $f'(x)$.

VI Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not. (5 pts ea)

a) $\lim_{x\to-\infty}\dfrac{\sqrt{3x^2-4}}{x+2}$ b) $\lim_{x\to-\infty}\dfrac{-2x+4}{9x^2+6}$ c) $\lim_{x\to\infty}\sqrt{x^2+x}-\sqrt{x^2+4}$

d) $\lim_{x\to-3}\dfrac{x^3+27}{9-x^2}$ e) $\lim_{x\to 9}\dfrac{6-2\sqrt{x}}{x-9}$ f) $\lim_{x\to 0^+} x\sqrt{\dfrac{4}{x^2}-1}$

g) $\lim_{x\to 0}\dfrac{|x|-x}{x}$ h) $\lim_{x\to -3}|-2|$ i) $\lim_{x\to 8^-}\dfrac{\sqrt{x}}{4}$

Extra Credit ---------- 5 pts ----------

Given $P>\sqrt{a}$ and $Q=\dfrac{P+\frac{a}{P}}{2}$. Prove: $P > Q$

I $f(x) = \dfrac{1}{\sqrt{6x^2+11x+3}}$

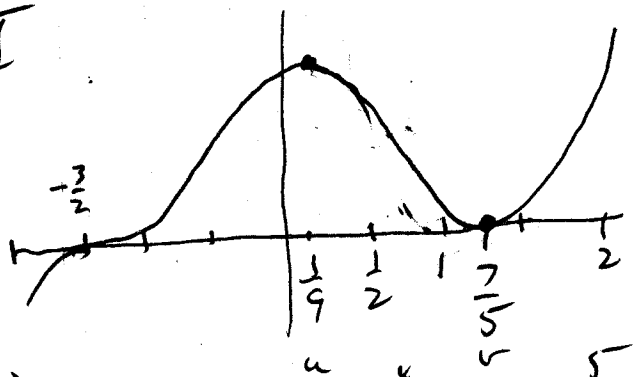
$(3x+1)(2x+3)$

D_f $+ \; 0 \; - \; 0 \; +$

$-\frac{3}{2} \quad -\frac{1}{3}$

$D_f \; x < -\frac{3}{2}$ or $x > -\frac{1}{3}$

II



Xmin −2
Xmax 2
Xscl .5
Ymin −200,000
Ymax 200,000
Yscl 100,000

b) $f(x) = (5x-7)^4 (2x+3)^5$

$u \qquad v$

$f'(x) =$

Advanced Placement Calculus

Name _____

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept!

First Exam

I What is the Domain ONLY of $f(x)=\dfrac{1}{\sqrt{6x^2+11x+3}}$ (5 pts)

II Given: $f(x) = (5x - 7)^4(2x + 3)^5$ (15 pts total)

a) Plot the graph of f on your grapher. Use an x–window from -2 to 2, and a y–window that is tall enough to fit the graph. Sketch the graph on your paper. Write the "window" settings you used.

b) Find the algebraic expression for $f'(x)$. Factor completely. Make a number line.

c) Determine the *coordinates* of all critical values. Show these points on your picture. The graph of f has a horizontal tangent line at each of these points.

d) Agree or disagree and support your position:

"The graph has a high point or a low point where $f'(x) = 0$."

III Solve for all $x \in \Re$. Graph solution on a number line. $\dfrac{|x+2|}{|x-3|} \geq 1$ (10 pts)

IV The "definition of the derivative" is generally written in one of two forms. Both forms require the use of limits. (15 pts total)

a) State **BOTH** of these "definitions."

b) Using one of your responses in part A, PROVE that $f'(-2)$ does not exist when $f(x) = |x - 2|$

c) Using the other of your responses in part A, find the derivative of $f(x)=2x^2+3$

V Given: $f(x) = |2x - 3| - |x - 2|$ (10 pts total)

a) Write $f(x)$ as a piece-meal function. b) Determine $f'(x)$.

VI Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not. (5 pts ea)

a) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{3x^2-4}}{x+2}$ b) $\lim\limits_{x\to\infty}\dfrac{-2x+4}{9x^2+6}$ c) $\lim\limits_{x\to\infty}\sqrt{x^2+x}-\sqrt{x^2+4}$

d) $\lim\limits_{x\to-3}\dfrac{x^3+27}{9-x^2}$ e) $\lim\limits_{x\to9}\dfrac{6-2\sqrt{x}}{x-9}$ f) $\lim\limits_{x\to0^+}x\sqrt{\dfrac{4}{x^2}-1}$

g) $\lim\limits_{x\to0}\dfrac{|x|-x}{x}$ h) $\lim\limits_{x\to-3}|-2|$ i) $\lim\limits_{x\to8^-}\dfrac{\sqrt{x}}{4}$

Extra Credit ---------- **5 pts** ----------

Given $P>\sqrt{a}$ and $Q=\dfrac{P+\dfrac{a}{P}}{2}$. Prove: $P > Q$

I $y = (2x-3)^2$

II $y = 4x^2 - 12x + 9$

$y' = 8x - 12$ set 4

$8x = 16$

$x = 2$

$y = (2 \cdot 2 - 3)^2$

$= 1$

3 pts

$y - 1 = 4(x-2)$

$y = 4x - 7$ set 0

$x = \frac{7}{4}$

x outwent.

II

B) $\frac{5}{1-x} < 3$

only one interval $-\infty$ – 5

I $5 < 3(1-x)$

$5 < 3 - 3x$

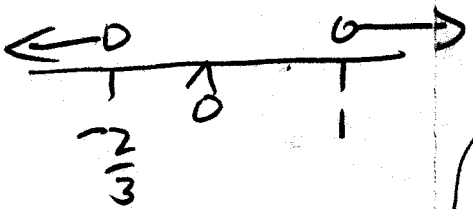
$3x < -2$

$x < -\frac{2}{3}$

II $5 > 3 - 3x$

$3x > -2$

$x > -\frac{2}{3}$



III $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

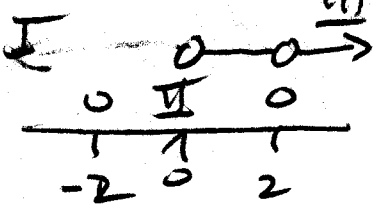
IV a) $t^2 = 5$

$t = 10$

b) $\frac{ds}{dt} = 2t \Big|_{t=10} = 20$

c) $\frac{\Delta s}{\Delta t} = 2$

4) $\left| \frac{x+2}{x-2} \right| \geq 1$



I $\frac{-(x+2)}{-(x-2)} \geq 1$ inequality reverses 2 pts for rev

$-x - 2 \geq -x + 2$

$-2 \geq 2$

Reject.

II $\frac{x+2}{-(x-2)} \geq 1$

$x + 2 \leq -x + 2$

$2x \leq 0$

$x \leq 0$

III $\frac{x+2}{x-2} \geq 1$

$x + 2 \geq x - 2$

$2 \geq -2$

True entire interval or $\mathbb{R}$.

$\lim_{h \to 0} \frac{1}{h} \left[\frac{1}{(x+h)^2+1} - \frac{1}{x^2+1} \right]$

$\lim_{h \to 0} \frac{1}{h} \left[\frac{x^2+1 - (x^2 - 2xh - h^2 + 1)}{((x+h)^2+1)(x^2+1)} \right]$

$\lim_{h \to 0} \frac{1}{h} \left[\frac{-2xh - h^2}{\text{etc}} \right]$

$\lim_{h \to 0} \frac{-2x - h}{((x+h)^2+1)(x^2+1)} = \frac{-2x}{(x^2+1)^2}$

V

a) $\lim_{x\to 0} \frac{x^3}{|x|} = 0$

$\lim_{x\to 0^-} \frac{x^3}{-x}$

$\lim_{x\to 0^-} x^2 = 0$

$\lim_{x\to 0^+} \frac{x^3}{x}$

$\lim_{x\to 0^+} x^2 = 0$

must show (W) (R)

b) $\lim_{x\to -1} \frac{x^2+4x+3}{x-1}$

$\frac{1-4+3}{-2} = 0$

c) $\lim_{x\to 3} \frac{x^3-27}{x-3}$

$\lim_{x\to 3} \frac{(x-3)(x^2+3x+9)}{x-3} = 27$

d) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x} \cdot \frac{\sqrt{\ }+\sqrt{\ }}{\sqrt{\ }+\sqrt{\ }}$

$\lim_{x\to 0} \frac{x+3-3}{x(\sqrt{x+3}+\sqrt{3})} = \frac{1}{2\sqrt{3}}$

e) $\lim_{x\to -\infty} \frac{\sqrt{x^2+2}}{3x-6}$

$\lim_{x\to -\infty} \frac{-x\sqrt{1+\frac{2}{x^2}}}{3x-6}$

$\lim_{x\to -\infty} \frac{-\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}} = -\frac{1}{3}$

f) $\lim_{x\to -1} (2+x)^{\frac{2}{x+1}}$

$\lim_{x\to 1} \left[(1+(1+x))^{\frac{1}{x+1}}\right]^2$

e^2 Noe -3

VI

$x+2y=2$

$y=-\frac{1}{2}x+1$

$m=-\frac{1}{2}$

$y=x^2+c$

$y'=2x \overset{\text{set}}{=} -\frac{1}{2}$

$x=-\frac{1}{4}$

$x+2y=2$

$-\frac{1}{4}+2y=2$

$2y=\frac{9}{4}$

$y=\frac{9}{8}$

$\frac{9}{8}=\frac{1}{16}+c$

$\frac{18}{16}+\frac{1}{16}=c$ $\quad c=\frac{17}{16}$.

Extra

2:03 pm

The rear of the train must travel $2\frac{1}{2}$ miles

$50\text{ mph} = \frac{5}{6}\text{ mile/min}$

$\frac{2.5\text{ miles}}{\frac{5}{6}\text{ mile/min}} = \frac{2.5\text{ miles}}{5\text{ miles}} \cdot 6\text{ min}$

$= 3$ minutes.

2:03.

1993

89 Velasco
84 Espino
78 Barnett
78 Moeser
74 Holm
72 Swegerton
66 Centeno
63 Self.

No changes.
excellent season

1994

Kate Russum 88
Greg Taylor 66
Erin Serris 58

Excellent Exam
Should have been
Very easy but
students really
not "into" it.
We did board work but
they seemed only interested
in the problem they did
rather than in all of
the problems—
Also impatient.
The bottoms 4 did
not take any Try! 1991

100	0
90	5
80	5
70	4
60	1
59	4

99

43

$\frac{1402}{19} = 73.8$

1997

100	0
90	3
80	5
70	4
60	5
59	2

97

77.9

34

as usual, The 34 didn't take any try (AWL
this (hopefully) is the
last of the lazy students.
(Next year might be better).
many want to discuss solutions
rather than come in with
them prepared...
this group is as weak as they
were last year. No changes
Needed. (In general exam...
change in students would
be a good idea)

A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!

Name 1ST Calc EXAM
Per _____ Date 97

Exam #1

I Find the x-intercept of the line tangent to $y = (2x - 3)^2$ when the slope is 4. (10 pts)

II Solve $\forall\ x \in \Re$. Graph the solution on a number line. (15 pts ea)

A) $\left|\frac{x+2}{x-2}\right| > 1$ B) $\frac{5}{1-x} < 3$

III Using the definition of the derivative, find $f'(x)$, given $f(x) = \frac{1}{x^2+1}$. (10 pts)

IV A car is parked 100 yards uphill from an intersection. At time $t = 0$ seconds, the car starts moving and rolls downhill, moving a distance $f(t) = t^2$ yards in t seconds. (10 pts tot)

A) When will the car reach the intesection?
B) What will the velocity of the car be when it reaches the intersection?
C) What is the acceleration while the car is moving?

V Limits. Determine the following limits, if possible. Be *very* clear concerning your method. (5 pts ea)

A) $\lim_{x\to 0} \frac{x^3}{|x|}$ B) $\lim_{x\to -1} \frac{x^2+4x+3}{x-1}$ C) $\lim_{x\to 3} \frac{x^3-27}{x-3}$

D) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$ E) $\lim_{x\to -\infty} \frac{\sqrt{x^2+2}}{3x-6}$ F) $\lim_{x\to -1} (2+x)^{\frac{2}{x+1}}$

VI Find the value of the constant c, such that the line $x + 2y = 2$ is tangent to the curve $y = x^2 + c$. (10 points)

Extra Credit ---------- 5 pts ----------

A train that is one-half mile long travels at a steady rate of fifty miles per hour. It enters a two-mile-long tunnel at exactly 2:00 P.M. At what time does the rear of the train emerge from the tunnel?

100	0
90	3
80	7
70	10
66	1
59	1

99

53

1741
~~1631~~ 22 = 79.1

53 to NewStudent - Not in my Trig/Anl class. concentrates but not prepared. answers are sloppy. Tag is on hand.

Ding Dong - wake up call.

~~change~~ ~~blog~~ delete "explains" from #11 maybe rephrase. these two pts are on the circle the radius is $\sqrt{20}$ - - -

97

100	0
90	7
80	4
70	4
60	0
59	2

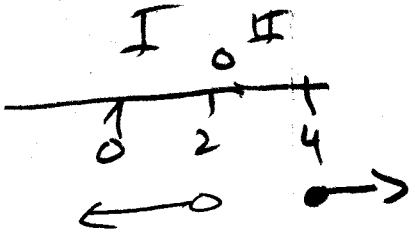
45

$\frac{1392}{17} = 81.9$

obviously the bottom two weren't in my trig/AnL class ~~last~~ year. I want them 86.3 are

I

a) $\frac{x}{x-2} \leq 2$



I $x \geq 2x-4$

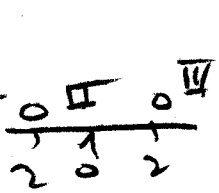
$4 \geq x$

II $x \leq 2x-4$

$4 \leq x$

$x < 2$ or $x \geq 4$

b) $\left|\frac{x-2}{x+2}\right| < 2$



I $\frac{x-2}{x+2} < 2$

$x-2 > 2x+4$

$-6 > x$

II $-\frac{(x-2)}{x+2} < 2$

$-x+2 < 2x+4$

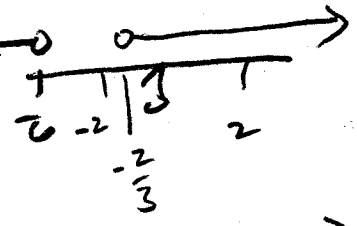
$-2 < 3x$

$-\frac{2}{3} < x$

III same as I

$-6 < x$

$\therefore x > 2$



$x < -6$ or $x > -\frac{2}{3}$

II $f(x) = |x+2| - |x-2|$

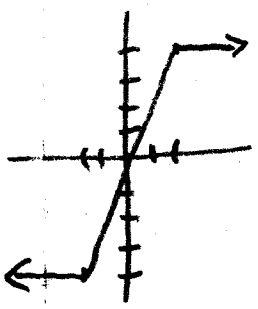
I $-x-2+x-2 = -4$

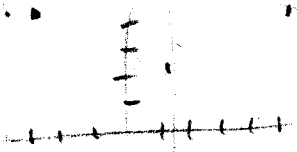
II $x+2+x-2 = 2x$

III $x+2-x+2 = 4$

$$f(x) = \begin{cases} -4 & \text{if } x < -2 \\ 2x & \text{if } -2 \leq x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$

$$f'(x) = \begin{cases} 0 & \text{if } x < -2 \\ 2 & \text{if } -2 < x < 2 \\ 0 & \text{if } x > 2 \end{cases}$$





III (1,2) (6,4)

$\sqrt{20}$

$d = \sqrt{(6-1)^2 + (4-2)^2}$

$\sqrt{20} \overset{\text{Sqr}}{=} \sqrt{b^2-2b+1+4}$

$20 = b^2-2b+5$

$b^2-2b-15 = 0$

$(b-5)(b+3) = 0$

$b \in \{-3, 5\}$

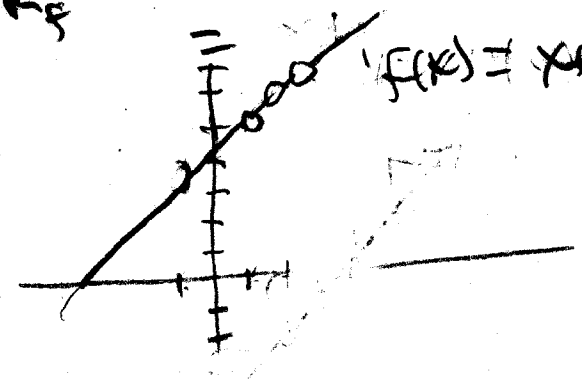
Visually is the intersection of $y=4$ and the circle with center at (1,2)

IV $f(x) = \frac{(x^2+3x-4)(x^2-5x+6)}{(x^2-3x+2)(x-3)}$

$D_f: x \neq \{1, 2, 3\}$

$\frac{(x+4)(x-1)(x-2)(x-3)}{(x-2)(x-1)(x-3)}$

$R_f: f(x) \neq 5$



IV $\lim_{h\to 0} \dfrac{x+h-\frac{1}{x+h}-\left(x-\frac{1}{x}\right)}{h}$

$\lim_{h\to 0} \frac{1}{h}\left(h-\frac{1}{x+h}+\frac{1}{x}\right)$

$\lim_{h\to 0} \frac{1}{h}\left(\frac{xh(x+h)-x+x+h}{x(x+h)}\right)$

$\lim_{h\to 0} \dfrac{x^2h+xh^2-x+x+h}{xh(x+h)}$

$\lim_{h\to 0} \dfrac{x^2+xh+1}{x(x+h)} = \dfrac{x^2+1}{x^2}$

VI D) $\lim_{x\to\infty} |-3| = 3$

E) $\lim_{x\to 8^-} \dfrac{\sqrt{x}}{4} = \dfrac{2\sqrt{2}}{4} = \dfrac{\sqrt{2}}{2}$

F) $\lim_{x\to\infty} \dfrac{\sqrt{x^2+2}}{3x-6}$

$\lim_{x\to\infty} \dfrac{x\sqrt{1+\frac{2}{x^2}}}{3x-6}$

$\lim_{x\to\infty} \dfrac{\sqrt{1+\frac{2}{x}}}{3-\frac{6}{x}} = \frac{1}{3}$

G) $\lim_{x\to -2} \dfrac{x^3+8}{x+2}$

$\lim_{x\to -2} \dfrac{(x+2)(x^2-2x+4)}{(x+2)}$

$4+4+4$

12

VII a) $\lim_{x\to 2} \dfrac{5x^3+4}{x-3}$

$\frac{44}{-1} = -44$

b) $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

$\lim_{x\to 2} \dfrac{(x-2)(x+2)}{x-2} = 4$

c) $\lim_{x\to\infty} \sqrt{\dfrac{3x+5}{6x-8}} =$

$\lim_{x\to\infty} \sqrt{\dfrac{3+\frac{5}{x}}{6-\frac{8}{x}}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

h) $\lim_{x\to 0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x} \cdot \dfrac{\sqrt{\ }+\sqrt{\ }}{\sqrt{\ }+\sqrt{\ }}$

$\lim_{x\to 0} \dfrac{x+3-3}{x(\sqrt{x+3}+\sqrt{3})}$

$\frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

Extra. $|x| < x^2+1$ $\quad \frac{0}{1}$ $\quad \frac{1}{0}$

I $-x < x^2+1$

$x^2+x+1 > 0$

$x = \dfrac{-1\pm\sqrt{1-4}}{2}$

x^2+x+1 Never $=0$

$x^2+x+1 > 0$ when $x=0$

$\therefore$ x^2+x+1 always >0

so original statement

II $x < x^2+1$

$x^2-x+1 > 0$

$x = \dfrac{1\pm\sqrt{1-4}}{2}$

same argument.

$\therefore$ True $\forall x$

Advanced Placement Calculus Name Calc 1st Exam

Copy original problem. Per _____ Date 98

Convince *me* that **you** understand the concept! 93

No Calculators.

Chapters 1 – 4.4 Exam

I Solve for x. Specifically state your solution and graph it on a number line (10 pts ea)

a) $\frac{x}{x-2} \le 2$

b) $\left|\frac{x-2}{x+2}\right| < 2$

II Write as a piecemeal function. Sketch $f(x)$. Determine $f'(x)$. (10 pts)

$f(x) = |x+2| - |x-2|$

III Point P with coordinates $(1,2)$ is connected to point Q with coordinates $(b, 4)$. The length between the points $\sqrt{20}$. Determine values of b. Explain. (10 pts)

IV Determine the domain and range of the function and draw a sketch. (10 pts)

$f(x) = \frac{(x^2+3x-4)(x^2-5x+6)}{(x^2-3x+2)(x-3)}$

V Using the $h \to 0$ version of the definition of the derivative, find $f'(x)$ given $f(x) = x - \frac{1}{x}$. (10 pts)

VI Evaluate the following limits, if they exist. Be sure your method is clean to me. If a limit does not exist, explain how you know that fact. (5 pts ea)

a) $\lim_{x \to 2} \frac{5x^3+4}{x-3}$

b) $\lim_{x \to 2} \frac{x^2-4}{x-2}$

c) $\lim_{x \to \infty} \sqrt{\frac{3x+5}{6x-8}}$

d) $\lim_{x \to \infty} |-3|$

e) $\lim_{x \to 8^-} \frac{\sqrt{x}}{4}$

f) $\lim_{x \to \infty} \frac{\sqrt{x^2+2}}{3x-6}$

g) $\lim_{x \to -2} \frac{x^3+8}{x+2}$

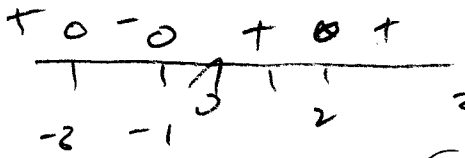
h) $\lim_{x \to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$

Extra Credit ---------- 5 pts ----------

Prove: $|x| < x^2 + 1$ for all $x \in \Re$.

I $f(x)=\sqrt{\dfrac{(x^2-x-2)}{x^2+x-6}}$

$=\sqrt{\dfrac{(x-2)(x+1)}{(x+3)(x-2)}}$



3 pts

D_f $x<-3$ or $x\geq -1$, $x\neq 2$

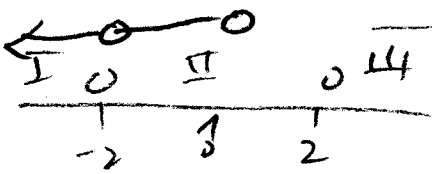
$x=2$ reducible Domain Restr.

II $f(x)=3x^4+2x^3$ $(-1,1)$

$f'(x)=12x^3+6x^2$ $y-1=-6(x+1)$

$f'(-1)=-12+6=-6=m$ $y=-6x-5$

III 4) $\dfrac{|x-2|}{|x+2|}>1$



I $\dfrac{-(x-2)}{-(x+2)}>1$

$-(x-2)>-(x+2)$

$-x+2>-x-2$

$2>-2$

true

II $-(x-2)>x+2$

$-x+2>$

$0>2x$

$0>x$

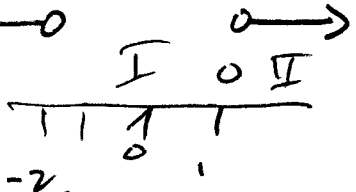
III $x-2>x+2$

$-2>2$

Reject,

III

b) $\dfrac{5}{1-x}<3$



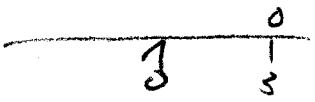
I $5<3-3x$

$3x<-2$

$x<-\frac{2}{3}$

II $x>-\frac{2}{3}$ must be in 2nd interval so $x>1$

c) $|2x+|x-3||<2$



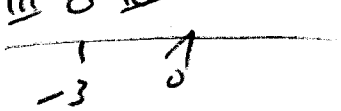
I

$|2x-(x-3)|<2$

$|2x-x+3|<2$

$|x+3|<2$

III o IV



III $-(x+3)<2$

$-x-3<2$

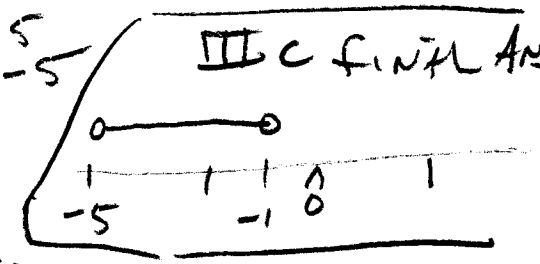
$-x<5$

$x>-5$

IV $x+3<2$

$x<-1$

III C final Ans



IV $f(x)=3x^2-3$

$\lim_{h\to 0}\dfrac{3(x+h)^2-3-(3x^2-3)}{h}$

$\lim_{h\to 0}\frac{1}{h}(3(x^2+2xh+h^2)-3-3x^2+3)$

$\lim_{h\to 0}\frac{3}{h}(x^2+2xh+h^2-x^2)$

$\lim_{h\to 0}3(2x+h)=6x=f'(x)$

II $|2x+x-3|<2$

$|3x-3|<2$

$3|x-1|<2$

$3x-3<2$

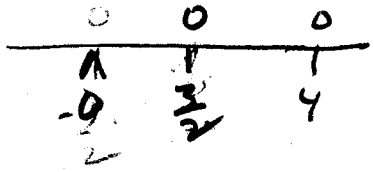
$3x<5$

$x<\frac{5}{3}$

Reject not in interval.

Since $x>3$

V $f(x) = |2x-3| - |4-x|$



I $-(2x-3) - (4-x)$

$-2x+3-4+x$

$-x-1$

II $2x-3-(4-x)$

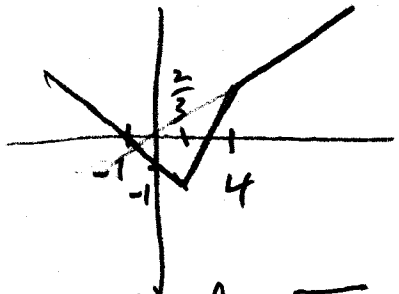
$2x-3-4+x$

$3x-7$

III $2x-3+4-x$

$x+1$

$$f(x) = \begin{cases} -x-1 \; ; \; x < \frac{3}{2} \\ 3x-7 \; ; \; \frac{3}{2} \le x < 4 \\ x+1 \; ; \; x \ge 4 \end{cases}$$



VI

a) $\lim_{x \to 3} \dfrac{\frac{1}{x} + \frac{1}{2}}{x-2}$

$\lim_{x \to 3} \dfrac{\frac{2+x}{2x}}{x-2}$

$\frac{5}{6}$

b) $\lim_{x \to -3} \dfrac{x^3+27}{x+3}$

$\lim_{x \to -3} \dfrac{(x+3)(x^2-3x+9)}{x+3}$

27

c) $\lim_{x \to 0} \dfrac{\sqrt{x+4}-2}{x} \dfrac{\sqrt{}+2}{\sqrt{}+2}$

$\lim_{x \to 0} \dfrac{x+4-4}{x(\sqrt{x+4}+2)} =$

$\frac{1}{2+2} = \frac{1}{4}$

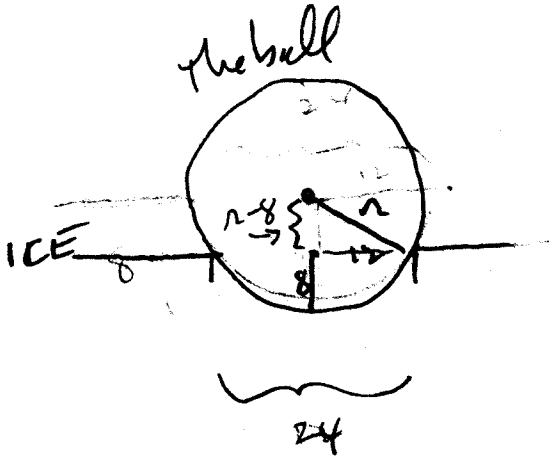
d) $\lim_{x \to 0} \dfrac{x^2 + \frac{1}{x}}{x^2 - \frac{1}{x}}$

$\lim_{x \to 0} \dfrac{\frac{x^3+1}{x}}{\frac{x^3-1}{x}} = -1$

e) $\lim_{x \to 0} (1-3x)^{\frac{2}{x}}$

$\lim_{x \to 0} \left[(1+(-3x))^{\frac{1}{-3x}} \right]^{-6}$

e^{-6}



$r^2 = (r-8)^2 + 12^2$

$r^2 = r^2 - 16r + 64 + 144$

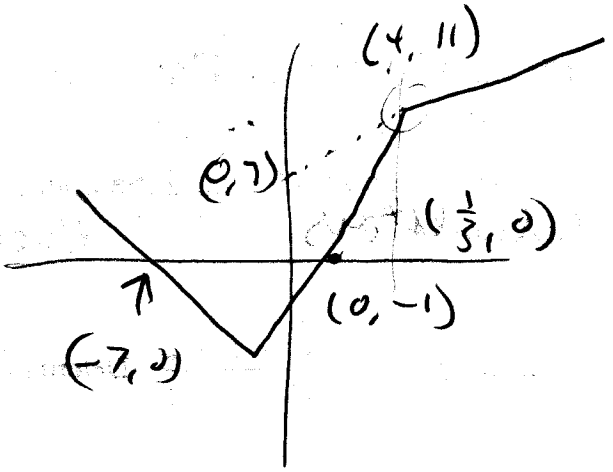
$16r = 208$

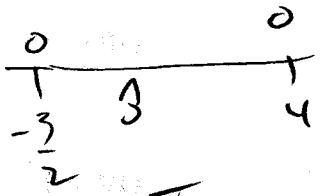
$r = 13,$

Ball radius 13".

V

as written





$|2x+3| - |4-x|$

II $-(2x+3) - (4-x)$

$-2x-3 + -4+x$

$-x-7$

III $2x+3-4+x$

$3x-1$

IV $2x+3+4-x$

$x+7$

$$f(x) = \begin{cases} -x-7 & \text{if } x < \frac{-2}{3} \\ 3x-1 & \text{if } -\frac{2}{3} \le x < 4 \\ x+7 & \text{if } x \ge 4 \end{cases}$$

actually, either plan II was fine.

100	1
90	4
80	7
70	3
60	1
54	1

101

51

$\frac{1380}{17}$ 81.76

as usual, the f didn't take any T/A class.

Advanced Placement Calculus Name CAL 1ST EXAM

Copy original problem. Per _____ Date 94, 99

Convince *me* that **you** understand the concept!

NO Calculators

Chapter 1 Exam

I Given $f(x) = \sqrt{\dfrac{x^2 - x - 2}{x^2 + x - 6}}$. Determine the domain of $f(x)$. Justify. (10 pts)

II Given $f(x) = 3x^4 + 2x^3$. Determine the ~~equation of~~ slope of the tangent line to $f(x)$ at $(-1, 1)$. (10 pts)

III Solve $\forall\ x \in \Re$. Graph solution set on a well labeled number line. Be especially sure that the reasons for your steps are very clear. (10 pts ea)

a) $\dfrac{|x-2|}{|x+2|} > 1$ b) $\dfrac{5}{1-x} < 3$ c) $|2x + |x - 3|| < 2$

IV Using the definition of the derivative which uses $h \to 0$, find $f'(x)$ given $f(x) = 3x^2 - 3$. (10 pts)

$f(x) \geq \frac{1}{2x+1}$

V Given $f(x) = |2x + 3| - |4 - x|$ (15 pts tot)

Should have been a negative sign with the inequality

a graph the solution to $f(x) \geq 8$

ba) Rewrite $f(x)$ as a piece-wise function.

Cb) Sketch the graph of $f(x)$.

VI Find the following limits: (5 pts ea)

a) $\lim\limits_{x \to 3} \dfrac{\frac{1}{x} + \frac{1}{2}}{x - 2}$ b) $\lim\limits_{x \to -3} \dfrac{x^3 + 27}{x + 3}$ c) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$

d) $\lim\limits_{x \to 0} \dfrac{x^2 + \frac{1}{x}}{x^2 - \frac{1}{x}}$ e) $\lim\limits_{x \to 0} (1 - 3x)^{\frac{2}{x}}$

Extra Credit ---------- 5 pts ----------

A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a hole 24 inches across the at the top and 8 inches deep. What was the radius of the ball?

I $f(x) = 3x^4 + 2x^3$

$f'(x) = 12x^3 + 6x^2 = -12 + 6 = -6$

$x = -1$

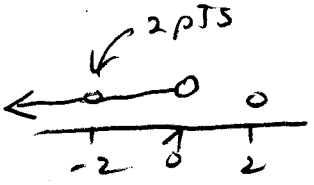
$y - 1 = -6(x+1)$

$y = -6x - 5 \overset{\text{set}}{=} 0$

$x = -\frac{5}{6}$

x intercept is $\left(-\frac{5}{6}, 0\right)$

II 4) $\left|\frac{x-2}{x+2}\right| > 1$ (2 pts)



I $\frac{x-2}{x+2} > 1$

$x - 2 < x + 2$

$-2 < 2$

True

II $\frac{-(x+2)}{x+2} > 1$

$-x + 2 > x + 2$

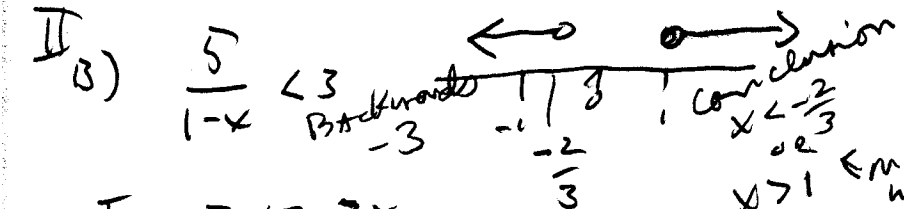
$0 > x$

III $\frac{x-2}{x+2} > 1$

$x - 2 > x + 2$

$-2 > 2$

reject.



II 3) $\frac{5}{1-x} < 3$ Backwards

<must have> conclusion $x < 0, x \neq -2$

I $5 < 3 - 3x$

$3x < -2$

$x < -\frac{2}{3}$

II $5 > 3 - 3x$

$3x > -2$

$x > -\frac{2}{3}$

AND in 2nd interval.

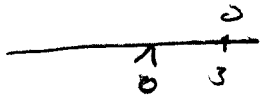
III $\lim_{h \to 0} \frac{\frac{1}{2x+2h+1} - \frac{1}{2x+1}}{h}$

$\lim_{h \to 0} \frac{1}{h}\left[\frac{2x+1-2x-2h-1}{(2x+2h+1)(2x+1)}\right]$

$\lim_{h \to 0} \frac{-2h}{h(2x+2h+1)(2x+1)}$

$\frac{-2}{(2x+1)^2}$

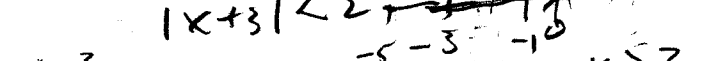
III c) $|2x + |x-3|| < 2$



I $x < 3$

$|2x - x + 3| < 2$

$|x + 3| < 2$



$x < -3$

$-x - 3 < 2$

$-x < 5$

$x > -5$

$-3 < x < 3$

$-x + 3 < 2$

$x < -1$

$x \geq 3$

$|2x + x - 3| < 2$

$|3x - 3| < 2$

$3x - 3 < 2$

$3x < 5$

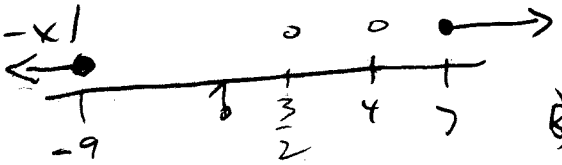
$x < \frac{5}{3}$

Reject not in interval.

conclusion $-5 < x < -1$

IV $f(x) = |2x-3| - |4-x|$

1) $|2x-3| - |4-x| \geq 8$



I $-(2x-3) - (4-x)$

$-2x + 3 - 4 + x$

$-x - 1 \geq 8$

$-9 \geq x$

II $-(2x-3) - (4-x)$

$2x - 3 - 4 + x$

$3x - 7 \geq 8$

$3x \geq 15$

$x \geq 5$

Reject not in interval.

III $2x - 3 + 4 - x \geq 8$

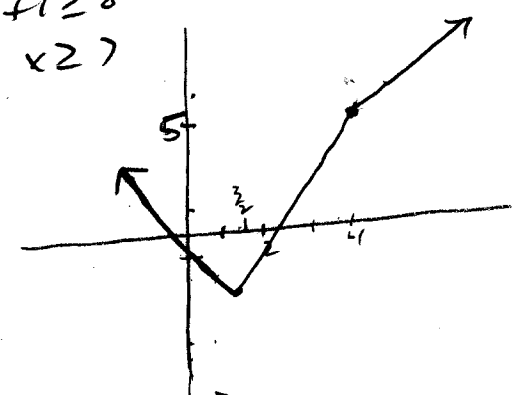
$x + 1 \geq 8$

$x \geq 7$

conclusion

$x \leq -9$ or $x \geq 7$

B) $f(x) = \begin{cases} -x - 1 & \text{if } x < \frac{3}{2} \\ 3x - 3 & \text{if } \frac{3}{2} \leq x < 4 \\ x + 1 & \text{if } x \geq 4 \end{cases}$



5

a) $\lim_{x \to 3} \frac{\frac{1}{x} + \frac{1}{2}}{x - 2}$

$\frac{\frac{1}{3} + \frac{1}{2}}{1}$

$\frac{5}{6}$

b) $\lim_{x \to -3} \frac{x^3 + 27}{x + 3}$

$\lim_{x \to -3} \frac{(x+3)(x^2 - 3x + 9)}{x+3}$

$9 + 9 + 9$

27

c) $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} \frac{\sqrt{} + 2}{\sqrt{} + 2}$

$\lim_{x \to 0} \frac{x + 4 - 4}{x(\sqrt{x+4} + 2)}$

$\lim_{x \to 0} \frac{1}{\sqrt{x+4} + 2}$

$\frac{1}{2+2}$

$\frac{1}{4}$

d) $\lim_{x \to 0} \frac{x^2 + \frac{1}{x}}{x^2 - \frac{1}{x}}$

$\lim_{x \to 0} \frac{x^3 + 1}{x^3 - 1}$

-1

e) $\lim_{x \to 0} (1 - 3x)^{\frac{2}{x}}$

$\lim_{x \to 0} \left[(1 + (-3x))^{\frac{1}{-3x}} \right]^{-6}$

$e^{-6} = \frac{1}{e^6}$

Extra $\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$

$\frac{2}{6} + \frac{1}{6}$

$\frac{3}{6}$

$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$

$= \frac{3}{12} + \frac{1}{12}$

$\frac{4}{12}$

$\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$

$\frac{4+1}{20}$

$\frac{5}{20}$

$\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$

$= \frac{5+1}{30}$

$\frac{6}{30}$

$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$

$= \frac{6+1}{42}$

$\frac{7}{42}$

9/24/99

100	2
90	4
80	11
70	7
60	0
59	0

101

71

2039

24

840

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators.

Name Calc

Per 1 S T. Date 9/9

EXAM 1,94

Chapter 1 Exam

I Given: $f(x)=3x^4+2x^3$. Determine the x-intercept of the tangent line to $f(x)$ at $(-1,1)$. (10 pts)

II Solve $\forall\ x\ \varepsilon\ \Re$. Graph the solution set on a *well labeled* numberline. Be especially sure that the reasons for your steps are very clear. (10 pts ea)

a) $\left|\frac{x-2}{x+2}\right|>1$ b) $\frac{5}{1-x}<3$ c) $|2x+|x-3||<2$

III Using the *definition of the derivative* which uses $h \to 0$, find $f'(x)$ given $f(x)=\frac{1}{2x+1}$. (10 pts)

IV Given: $f(x)=|2x-3|-|4-x|$ (25 pts tot)

a) Graph the solution to $f(x)\geq 8$.

b) Rewrite $f(x)$ as a piece-wise function.

c) Sketch the graph of $f(x)$ on a standard cartesian axis.

V Find the following limits (5 pts ea)

a) $\lim_{x\to 3}\frac{\frac{1}{x}+\frac{1}{2}}{x-2}$ b) $\lim_{x\to -3}\frac{x^3+27}{x+3}$ c) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$

d) $\lim_{x\to 0}\frac{x^2+\frac{1}{x}}{x^2-\frac{1}{x}}$ e) $\lim_{x\to 0}(1-3x)^{\frac{2}{x}}$

Extra Credit ---------- **5 pts** ----------

Confirm that: $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$ and $\frac{1}{3}=\frac{1}{4}+\frac{1}{12}$.

Represent these three fractions in a similar manner: $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

Calculus 1st exam

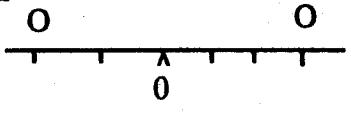
Absolute Bliss

Mr. Mumaugh

I Your job is to supply **detailed explanations** for each of the parts A -> E. Copy the problem down the left hand side of your paper. On the right hand side, write your explanations and justifications.

Given: $\left|\frac{x-3}{x+2}\right| < \frac{1}{2}$

A



B

$$\frac{x-3}{x+2} < \frac{1}{2}$$
$$2x-6 > x+2$$
$$x > 8$$

Reject because ...

C

$$\frac{-(x-3)}{x+2} < \frac{1}{2}$$
$$-2x+6 < x+2$$
$$4 < 3x$$
$$\frac{4}{3} < x$$

D

$$\frac{x-3}{x+2} < \frac{1}{2}$$
$$2x-6 < x+2$$
$$x < 8$$

E conclusion

II Solve and graph on a number line

A) $|x+3|-|x-2|<3$ B) $|x|+|x|+|x+1|>5$ C) $\left|\frac{x+2}{x-3}\right| < \frac{1}{2}$

III Re-write each of the following functions as "piece-meal" functions. Graph on the cartesian plane.

A) $F(x)=|2x+3|$ B) $F(x)=|x|+|x+1|$ C) $F(x)=|2x-1|-|2x+1|$ D) $F(x)=\left|x^2+2\right|-\left|x^2-1\right|$

IV

1) The solution set of: $\frac{x+3}{x-2} \le \frac{x+2}{x-2}$ is

a) $x \in \Re$ b) $x \in \Re,\ x \neq 2$ c) $x<2$ d) $x>2$ e) $x \in \{\ \}$

2) What values of x satisfy the inequality: $\frac{x-1}{x} > \frac{1}{2}$?

a) $x>2$ b) $x<2$ c) $x>\frac{1}{2}$ d) $x<\frac{1}{2}$ e) $x<0$ or $x>2$

3) The solution set of the inequality $|-5x+3| \le 2$ is:

a) $(-\infty, \frac{1}{5}] \cup [1, \infty)$ b) $[\frac{1}{5},\ 1]$ c) $[-1, 1]$ d) $(-\infty, -1] \cup [1, \infty)$ e) $(-\infty, -1] \cup [-\frac{1}{5}, \infty)$

4) The solution set of the inequality $\frac{4x^3-8x^2-3x+9}{(x+1)^3} \ge 0$ is:

a) $[-1, \frac{3}{2}]$ b) $(-1, \frac{3}{2}]$ c) $(-\infty, -1) \cup [\frac{7}{3}, \infty)$ d) $(-\infty, -1) \cup (-1, \infty)$ e) $[\frac{3}{2}, \infty)$

5) What values of x make the inequality $|x-2|+x>1$ true?

a) $(2, \infty)$ b) $(-2, \infty)$ c) $(-\infty, \infty)$ d) $(\frac{3}{2}, \infty)$ e) $(-\infty, -\frac{3}{2})$

6) If $|2x-3|<\varepsilon$ then the solution for x in terms of ε is

a) $\frac{3-\varepsilon}{2} < x < \frac{3+\varepsilon}{2}$ b) $x < \frac{3}{2}+\varepsilon$ c) $x < \frac{3}{2}+\frac{\varepsilon}{2}$ d) $x < \frac{\varepsilon}{2}$ e) $\frac{3-\varepsilon}{2} < x < 3+\frac{\varepsilon}{2}$

V Absolute Bliss (continued)

1) The equation of a line perpendicular to $x-3=5$ and passing through the point $(2, 1)$ is

a) $x=2$ b) $y=1$ c) $x=8$ d) $y=2$ e) $x=1$

2) In the linear equation $ax+by=c$ which of the following is necessarily true?

I. The slope of the line is $-\frac{b}{a}$

II. The y-intercept is $\frac{c}{a}$

III. The line contains the point $\left(d, \frac{c-ad}{b}\right)$

a) I only b) II only c) III only d) I and II only e) II and III only

3) The slope of a line perpendicular to the line through the points $(3, -5)$ and $(-2, -4)$ is

a) $-\frac{1}{5}$ b) $\frac{1}{5}$ c) -9 d) $-\frac{1}{9}$ e) 5

4) Find the value of k such that the lines with equations $4kx-2y=1$ and $3y+2kx=-5$ are parallel.

a) $-\frac{2}{3}$ b) 2 c) $\{\}$ d) zero

5) The length of the longest side of a triangle with verticies (2, 3), (2,1) and (5, 4) is:

a) 2 b) $\sqrt{2}$ c) $3\sqrt{2}$ d) $\sqrt{10}$ e) $\sqrt{19}$

6) Find the value of b so that $(3, b)$, (4, 1), and (–2, 5) are collinear.

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 4 d) $\frac{5}{3}$ e) $-\frac{3}{4}$

7) Two lines $ax+by=3$ and $bx+ay=2$ intersect at (4,-1) if $a=$

a) $\frac{11}{15}$ b) $\frac{14}{15}$ c) $\frac{17}{30}$ d) $\frac{23}{30}$ e) $\frac{5}{6}$

8) If $f(x)=x^2+3x$ and $g(x)=x^3-x$ then $(f\circ g)(2)=$

a) 990 b) 60 c) 45 d) 54 e) 8

9) Let $f(x)=|3x-2|+2|x|-5$. For values of x in the interval $\left(0, \frac{2}{3}\right)$, $f(x)=$ ...

a) $-x-3$ b) $5x-7$ c) $-5x-7$ d) $x-7$ e) $x+3$

10) The domain of the function $f(x)=\dfrac{3x}{\sqrt{4-x^2}}$ is:

a) $[-2, 2]$ b) $(-2, 2)$ c) $(-2, 2)\,;\, x\neq 0$ d) $[-2, 2]\,;\, x\neq 0$ e) $|x|>2$

11) Which of the following functions is equivalent to $f(x)=\dfrac{|x-2|}{x-2}$?

a) $f(x)=1$ b) $f(x)=\begin{cases}-1 & \text{if } x<2\\ 1 & \text{if } x\geq 2\end{cases}$ c) $f(x)=\begin{cases}-1 & \text{if } x\leq 2\\ 1 & \text{if } x>2\end{cases}$

d) $f(x)=\begin{cases}-1 & \text{if } x<2\\ 1 & \text{if } x>2\end{cases}$ e) $f(x)=1,\ x\neq 2$

VII For the following problems, you are required to write up your answers in a form which includes an explanation of your methods. For example, in problem 20 part **d** *how* did you actually figure the estimate the instantaneous rate of change of price?

1) Board Prices. If you check the prices of various lengths of lumber you will find that a board twice a long as another of the same type does not necessarily cost twice as much. Let x be the number of feet long a 2×6 board is and let y be the number of cents you pay for the board. Assume that y is given by

$y = 0.2x^3 - 4.8x^2 + 80x$.

a) Find the price of a 2×6 that is 5 feet long, 10 feet long, and 20 feet long.

b) Find the average rate of change of the price, cents per foot, for 5 to 5.1 feet, 5 to 5.01 feet, and 5 to 5.001 feet.

c) The average number of cent per foot in part (b) is approaching an *integer* as the change in x gets smaller and smaller. What integer? What name would you give this rate of change?

d) Estimate the instantaneous rate of change of price if x is 10 feet and if x is 20 feet. You should find that each of these rates is an integer.

e) One of the principles of marketing is that when you buy in larger quantities, you usually pay less per unit. Explain how the numbers in this problem show that this principle does *not* apply to buying longer boards. Why does it not apply?

2) The object of this problem is to determine the effect on the graph of $y = f(x)$ of changing the parameters A, B, C, and/or D in the function $g(x) = A \cdot f(Bx + C) + D$. How does the new function g relate to the original function f when one or more of the parameters A, B. C, or D is changed? In particular, how are the domain, range, and symmetries of a function changed when the function f is changed in g ? How does the shape of the graph of the function change? Most importantly, what effect does each of these parameters have on a function f that causes these changes to occur?

A) Graph each of the following functions, with appropriate domains and ranges. Records the results with paper and pencil, labeling the graphs and including the scales of both axes.

function	$y = f(x)$	f is the parent function of:	Symmetric with respect to: y-axis	origin
1.	$y = x$	$y = mx + b$		
2.	$y = x^2$	$y = ax^2 + bx + c$		
3.	$y = \|x\|$	$y = a\|bx + c\| + d$		
4.	$y = \sin x$	$y = a \sin(bx + c) + d$		
5.	$y = \frac{1}{x}$	$y = \frac{a}{(bx + c)} + d$		

B) Using the functions 3, 4 and 5 in part (a), investigate the effect of adding a constant to the function $y = f(x)$.

a) Graph the following: i. $f(x) = |x|$ ii. $g(x) = |x| + 3$ iii. $g(x) = |x| - 4$

Graph $f(x) = |x|$ on [–10,10] by [-6, 6]. Overlay the graph $g(x) = |x| + 3$. (What is D?) Overlay the graph $g(x) = |x| - 4$ on the same viewing rectangle (D = – 4 this time).

b) Repeat part (a) for functions 4 and 5, graphing $y = f(x)$, $g(x) = f(x) + 3$, and $g(x) = f(x) - 4$ in each case. Clear the graphics screen before graphing each new set of related functions.

c) How does the change in D effect the graph of f?
Describe the shape and position of the graph considering D.

C) Using functions 3, 4, and 5 from part A, investigate the effect of changing the parameter C in the function $g(x) = Af(Bx + C) + D$.

a) For each of the functions 3 and 5 in part (A) compare the graphs of the following functions. Using an appropriate domain and range, sketch all three functions on the same viewing rectangle.

i. $y = f(x)$ ii. $g(x) = f(x+2)$ iii. $g(x) = f(x-4)$

b) Using function 4, graph $y = f(x+\pi)$, $y = f\left(x+\frac{\pi}{2}\right)$, $y = f\left(x-\frac{\pi}{2}\right)$ together on the viewing rectangle [−6.3, 6.3] by [− 1.5, 1.5]. What familiar trigonometric functions (if any) do these graphs resemble?

c) Considering the results from observations made in parts a and b, describe the shape and position of the graph $g(x) = f(x+C)$ when: i. $C > 0$ ii. $C < 0$

d) Why does the graph of g appear as it does as compared to the graph of f. Describe the effect of first adding C to x then performing the function on this quantity.

3) Graph $f(x) = \dfrac{x^2-3x-4}{x^2-1}$ on [−2, 2] by [−10, 10] by putting $f(x)$ into Y_1. Note that $y = f(x)$ undefined at $x = -1$ and $x = 1$.

A) From the graph, how does f appear tobehave near $x = -1$? near $x = 1$?

B) For values of x (1 or − 1) where the graph appears to be continuous (has no visib le holes), use the trace option of the graphics utility to estimate the valy of y for this choice of x.

C) To determine the behavior of $f(x) = \dfrac{x^2-3x-4}{x^2-1}$ for x near 1 numerically, complete the following table of values. Use the program BEHAVIOR. To complete the first two columns, let H = .5. For the last two columns, let H = − .5 In this part use A = 1. (In part F below use A = − 1.) Notice that for each successive table value, the value of H is half that of the previous entry for H.

H	A + H	f(A + H)	A − H	f(A − H)
0.5	1.5	−5	0.5	7
0.25	1.25	−11	0.75	13
0.125			0.875	
0.0625			0.9375	
0.03125				
0.015625				
0.0078125				
0.00390625				
0.001953125				

```
Program; New; <Enter>
BEHAVIOR  <Enter>
Prompt A
Prompt H
Lbl 1
A + H store X
Disp "X"
Disp X
Disp "Y1"
Disp Y1
Pause
H/2 store H
Goto 1
```

D) Observe the magnitude of successive values of y in the table above. Describe the behavior of $f(x) = \dfrac{x^2-3x-4}{x^2-1}$ for x near 1.

E) Compare the values in the table to the graph of . How are these related? Explain.

F) To determine the behavior of $y = f(x)$ for values of x near − 1, complete the table below.

G) Compare the successive y–values listed in part F to graph of f. Explain how these are related.

H	A + H	f(A + H)	A − H	f(A − H)
0.5	−0.5	3	−1.5	2.2
0.25	−0.75	2.714285714	−1.25	2.3333333333
0.125			−1.125	
0.0625			−1.0625	
0.03125				
0.015625				
0.0078125				
0.00390625				
0.001953125				

Absolute Bliss VI

Mr. Mumaugh

Define: $\operatorname{sgn} * = \begin{cases} -1 & \text{if } * < 0 \\ 0 & \text{if } * = 0 \\ 1 & \text{if } * > 0 \end{cases}$ the function $\ln *$ is defined for $*>0$

Determine the domain of the composition of each of the following combinations of equations. You should not compute the actual composition

$$a(x)=(x+2)(x-1) \qquad b(x)=(x-2)(x+1) \qquad c(x)=\frac{x-3}{x^2-6x+9}$$

$$d(x)=\frac{x+3}{x^2+3x+2} \qquad e(x)=\frac{x^2+10x+25}{x+5} \qquad f(x)=\frac{x^2-25}{x+5}$$

A) $A(x)=\dfrac{c(x)}{d(x)}$ B) $B(x)=d(c(x))$ C) $C(x)=c(c(x))$ D) $D(x)=\sqrt{c(x)}$

E) $E(x)=\sqrt{d(x)}$ F) $F(x)=\dfrac{d(x)}{c(x)}$ G) $G(x)=\sqrt{\dfrac{1}{d(x)}}$ H) $H(x)=d(d(x))$

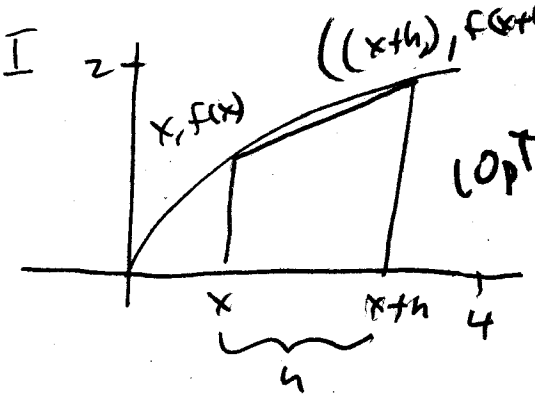
I) $I(x)=\dfrac{\sqrt{c(x)}}{d(x)}$ j) $J(x)=\dfrac{\sqrt{c(x)}}{\sqrt{d(x)}}$ K) $K(x)=\sqrt{\dfrac{c(x)}{d(x)}}$ L) $L(x)=\sqrt{\operatorname{sgn}(a(x))}$

M) $M(x)=\sqrt{\operatorname{sgn}(b(x))}$ N) $N(x)=\ln(a(x))$ O) $O(x)=\ln(d(x))$ P) $P(x)=\ln\left(\dfrac{c(x)}{d(x)}\right)$

Q) $Q(x)=d(e(x))$ R) $R(x)=e(a(x))$ S) $S(x)=e(e(x))$ T) $T(x)=e(f(x))$

U) $U(x)=f(f(x))$ V) $V(x)=f(e(x))$ W) $W(x)=d(c(c(x)))$ X) $X(x)=\sqrt{a(x)+b(x)}$

I



10 pts

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{f(x+h) - f(x)}{x+h - x}$

$m = \frac{f(x+h) - f(x)}{h}$

as $h \to 0$, h, The distance between The 2 x values decreases & The point That x and x+h are The same point. the slope of the "chord" becomes a tangent line at $(x, f(x))$.

II $(-3, 2k)$ $(5k, 7)$

$\frac{7-2k}{5k+3}$ set $= 6$

$7 - 2k = 30k + 18$

$-11 = 32k$

$k = \frac{-11}{32}$ 10 pts

III $f(x) = \frac{4}{x}$

A) $\lim_{h \to 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h}$

$\lim_{h \to 0} \frac{4x - 4x - 4h}{xh(x+h)}$

$\lim_{h \to 0} \frac{-4}{x(x+h)} = -\frac{4}{x^2}$

10

B) $f'(x) = -\frac{4}{x^2}\Big|_{x=2} = -1$

$1 \cdot m = 1$

$f(2) = 2, (2, 2)$

$y - 2 = 1(x - 2)$

$y = x$ 10

IV

A) $\lim_{x \to 0} f(x) = 0$ $\lim_{x \to 0} g(x) = 0$ $\lim_{x \to 0} \frac{f}{g} = 1$ $f = x$ $g = x$ 5

B) $\lim_{x \to 0} f(x) = 0$ $\lim_{x \to 0} g(x) = \infty$ $\lim_{x \to \infty} fg = 1$ $f = x$ $g = \frac{1}{x}$ 5

C) $\lim_{x \to 0} f(x) = 0$ $\lim_{x \to 0} g(x) = \infty$ $\lim_{x \to 0} fg = 0$ $f = x^2$ $g = \frac{1}{x}$ 5

D) $\lim_{x \to 0} f(x) = 4$ $\lim_{x \to 0} g(x) = \frac{1}{2}$ $\lim_{x \to 0} fg = 2$ $f = 4$ $g = \frac{1}{2}$ 5

U

A) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3-x}}{x} \cdot \frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}$

$\lim_{x\to 0} \frac{3+x-3+x}{x(\sqrt{3+x}+\sqrt{3-x})}$ ✓

$\lim_{x\to 0} \frac{2}{\sqrt{3+x}+\sqrt{3-x}} = \frac{2}{2\sqrt{3}} = \frac{\sqrt{3}}{3}$

B) $\lim_{x\to 1} \frac{(x+1)^3}{(x+1)^5}$

$\lim_{x\to 1} \frac{1}{(x+1)^2} = \frac{1}{4}$ ✓

C) $\lim_{x\to 2} \frac{2\sqrt{x+1}}{x+2}$

$\frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ ✓

D) $\lim_{x\to 2^-} \frac{4-x^2}{\sqrt{2+x-x^2}} \cdot \frac{\sqrt{}}{\sqrt{}}$

$\lim_{x\to 2^-} \frac{(2-x)(2+x)\sqrt{2+x-x^2}}{(2-x)(1+x)}$

$\frac{4(0)}{3} = 0$ ✓

E) $\lim_{x\to 9} \frac{6-2\sqrt{x}}{x-9} \cdot \frac{6+2\sqrt{x}}{6+2\sqrt{x}}$

$\lim_{x\to 9} \frac{36-4x}{(x-9)(6+2\sqrt{x})}$

$\lim_{x\to 9} \frac{4(9-x)}{(x-9)(6+2\sqrt{x})}$ ✓

$\frac{-4}{6+6} = \frac{-4}{12} = -\frac{1}{3}$

F) $\lim_{h\to 0} \frac{(1+h)^{\frac{1}{3}}-1}{h}$

def of der of $f(x) = (x+1)^{\frac{1}{3}}$

at $x=0$

$\therefore f'(x) = \frac{1}{3}(x+1)^{-\frac{2}{3}}\Big|_{x=0} = \frac{1}{3}$ ✓

G) $\lim_{x\to 0^-} \frac{x}{\sqrt{x^3+x^2}}$

$\lim_{x\to 0^-} \frac{x}{|x|\sqrt{x+1}}$

$\lim_{x\to 0^-} \frac{-1}{\sqrt{x+1}} = -1$ ✓

H) $\lim_{x\to 1} \frac{x^3-x^2+2x-2}{x-1}$

$\lim_{x\to 1} \frac{x^2(x-1)+2(x-1)}{x-1}$

$\lim_{x\to 1} x^2+2 = 3$ ✓

9/30/85 76
69

10-3-89 Mark Manning 93
Phillip Walker F

10-4-90 Staci Ellis 94
Carla Land 93
Gayle Moore 85
Sabina Leung 83

10/4/95

Roxanne Clark 90
Gabe Warden 86
Irene Dutra 86
Chrystelle Ocampo 76
Casey Anderson 69
Nathan Hughes 69

9/28/2000

Dustin Dellon 94
Mike Tuel 82
Sunita Chamba 62

10/3/2002 Steve Norris 87
Mimi Nau 78
Jacob Dimante 77
Jessica Ritter 74
Matthew Bowman 66
Kristen Jones 66
Candice Hansen 44

A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!

Name EXAM #1 MU

Per _____ Date _____

Exam # 1 Make-Up

I Draw the graph of $F(x)=\sqrt{x}$ for $x \in [0,4]$. Locate some point, x, on the x-axis and another point, $x+h$, $(h>0)$ also on the x-axis. Illustrate that: $\dfrac{F(x+h)-F(x)}{h}$ is, indeed, the slope of a line. **(Explain very clearly)**. Furthermore, explain how applying the limit as h goes to zero will result in the slope of the tangent line at the point $(x,F(x))$. (10 pts)

II Find k so that the line joining the points with coordinates: $(-3,2k)$ and $(5k,7)$ has slope 6. (10 pts)

III Given: $F(x)=4x^{-1}$ (10 pts ea)

A) Using the definition of the derivative, find $F'(x)$.

B) Find the equation of the line perpendicular to the tangent line to $F(x)$ at $x=2$.

IV Choose functions $F(x)$ and $G(x)$ such that they satisfy each of the following: (Parts A, B, C, and D are independent of each other. That is, each is different problem.) (5 pts ea)

A) $\lim_{x\to 0} F(x)=0$ $\lim_{x\to 0} G(x)=0$ $\lim_{x\to 0} \dfrac{F(x)}{G(x)}=1$

B) $\lim_{x\to 0} F(x)=0$ $\lim_{x\to 0} G(x)=\infty$ $\lim_{x\to 0} F(x)G(x)=1$

C) $\lim_{x\to 0} F(x)=0$ $\lim_{x\to 0} G(x)=\infty$ $\lim_{x\to 0} F(x)G(x)=0$

D) $\lim_{x\to 0} F(x)=4$ $\lim_{x\to 0} G(x)=\dfrac{1}{2}$ $\lim_{x\to 0} F(x)G(x)=2$

V Limits. Evaluate the following limits. Be sure your method is clear to me. If a limit does not exist, explain why it does not. (5 pts ea)

A) $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3-x}}{x}$

B) $\lim_{x\to 1} \dfrac{(x+1)^3}{(x+1)^5}$

C) $\lim_{x\to 2} \dfrac{2\sqrt{x+1}}{x+2}$

D) $\lim_{x\to 2^-} \dfrac{4-x^2}{\sqrt{2+x-x^2}}$

E) $\lim_{x\to 9} \dfrac{6-2\sqrt{x}}{x-9}$

F) $\lim_{h\to 0} \dfrac{(1+h)^{\frac{1}{3}}-1}{h}$

G) $\lim_{x\to 0^-} \dfrac{x}{\sqrt{x^3+x^2}}$

F) $\lim_{x\to 1} \dfrac{x^3-x^2+2x-2}{x-1}$

10/2/97

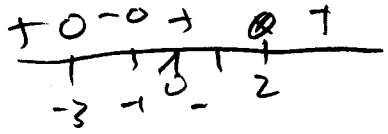
Trevor Hughes 84
Ian Scudder 84
Katy green 70
troy fonte 65
mike Evers 59

9/26/01

Amie Gutierrez 94
Valerie Cole 93
Kristen Suchkomen 91
Jenny Bitts-Houck 88
Ashley Centers 78
Brittany Gonzales 75
Lang Tang 68
Dustin Davis 45

I $f(x) = \sqrt{\frac{x^2-x-2}{x^2+x-6}}$

$= \sqrt{\frac{(x-2)(x+1)}{(x-2)(x+3)}}$



Dom $x < -3$ or $x \geq -1$; $x \neq 2$

$x = 2$ is a removable domain restriction.

II $f(x) = 3x^4 + 2x^3 \quad (-1, 1)$

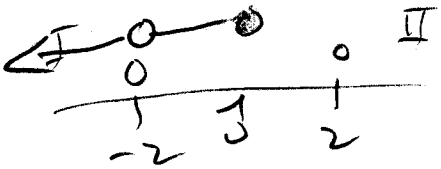
$f'(x) = 12x^3 + 6x^2$

$f'(1) = -12 + 6 = -6 = m$

$y - 1 = -6(x+1)$

$y = -6x - 5$

III A) $\left|\frac{x-2}{x+2}\right| \geq 1$



I $\frac{x-2}{x+2} \geq 1$

$x - 2 < x + 2$

$-2 < 2$

true

II $-\frac{(x-2)}{x+2} \geq 1$

$-x + 2 \geq x + 2$

$0 \geq x$

III $\frac{x-2}{x+2} \geq 1$

$x - 2 \geq x + 2$

$-2 \geq 2$

rejected.

so $x < 0$; $x \neq -2$

III B) $\frac{5}{1-x} < 3$

I $5 < 3 - 3x$

$3x < -2$

$x < -\frac{2}{3}$

II $5 > 3 - 3x$

$3x > -2$

$x > -\frac{2}{3}$

$x < -\frac{2}{3}$ or $x > 1$

IV $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ \sqrt{x} - 1 & \text{if } x \geq 1 \end{cases}$

$\lim_{x \to 1} \frac{f(x) - 0}{x - 1}$

$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1}$

$\lim_{x \to 1^-} \frac{(x+1)(x-1)}{(x-1)}$

2

$\lim_{x \to 1^+} \frac{\sqrt{x} - 1}{x - 1}$

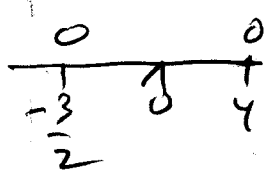
$\lim_{x \to 1^+} \frac{\sqrt{x} - 1}{(\sqrt{x} - 1)(\sqrt{x} + 1)}$

$\frac{1}{2}$

Since $2 \neq \frac{1}{2}$ limit DNE

(L) $\neq$ (R) Since that limit is Def of The Der, Then DNE at $x = 1$

V $f(x) = |2x+3| - |4-x|$



I $-(2x+3) - (4-x)$

$-2x-3-4+x$

$-x-7$

II $2x+3-(4-x)$

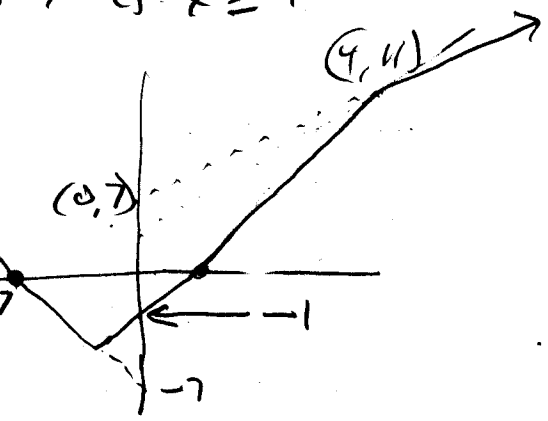
$2x+3-4+x$

$3x-1$

III $2x+3+4-x$

$x+7$

$$f(x) = \begin{cases} -x-7 & \text{if } x < -\frac{3}{2} \\ 3x-1 & \text{if } -\frac{3}{2} \le x < 4 \\ x+7 & \text{if } x \ge 4 \end{cases}$$



$$f'(x) = \begin{cases} -1 & \text{if } x < -\frac{3}{2} \\ 3 & \text{if } -\frac{3}{2} < x < 4 \\ 1 & \text{if } x > 4 \end{cases}$$

VI

a) $\lim_{x \to 0^-} \frac{2x^2+3x}{|x|}$

$\lim_{x \to 0^-} -(2x+3)$

-3

b) $\lim_{x \to -3} \frac{x^3+27}{x+3}$

$\lim_{x \to -3} \frac{(x+3)(x^2-3x+9)}{x+3}$

$9+9+9$

27

c) $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x} \frac{\sqrt{}+2}{\sqrt{}+2}$

$\lim_{x \to 0} \frac{x+4-4}{x(\sqrt{}+2)}$

$\lim_{x \to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{2+2}$

$= \frac{1}{4}$

d) $\lim_{x \to 0} \frac{(x^2+\frac{1}{x})x}{(x^2-\frac{1}{x})x}$

$\lim_{x \to 0} \frac{x^3+1}{x^3-1}$

$\frac{1}{-1} = -1$

e) $\lim_{x \to 0} (1-3x)^{\frac{2}{x}}$

$\lim_{x \to 0} \left[(1+(-3x))^{\frac{1}{-3x}} \right]^{-6}$

e^{-6}

f) $\lim_{x \to \infty} \sqrt{x^2+x} - \sqrt{x^2+4} \frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}$

$\lim_{x \to \infty} \frac{x^2+x-x^2-4}{\sqrt{} + \sqrt{}}$

$\lim_{x \to \infty} \frac{x(1-\frac{4}{x})}{x(\sqrt{1+\frac{1}{x}} + \sqrt{1+\frac{4}{x}})} = \frac{1}{1+1} = \frac{1}{2}$

g) $\lim_{x \to 1} f(x)$

$$g(x) = \begin{cases} 4-x & \text{if } x < 1 \\ \sqrt{6} & \text{if } x = 1 \\ 2+x^2 & \text{if } x > 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = 4-1 = 3$

$\lim_{x \to 1^+} f(x) = 2+1^2 = 3$

$\therefore \lim_{x \to 1} f(x) = 3$

Advanced Placement Calculus

Name Calc 1ST Exam

Per _____ Date Make up

Copy original problem.

Convince *me* that you understand the concept!

Chapter 1 Make – Up Exam

I Given $f(x) = \sqrt{\dfrac{x^2 - x - 2}{x^2 + x - 6}}$. Determine the domain of $f(x)$. Justify. (10 pts)

II Given $f(x) = 3x^4 + 2x^3$. Determine the equation of the tangent line to $f(x)$ at $(-1, 1)$. (10 pts)

III Solve $\forall\ x \in \Re$. Graph solution set on a well labeled number line. Be especially sure that the reasons for your steps are very clear. (10 pts ea)

a) $\left|\dfrac{x-2}{x+2}\right| > 1$ b) $\dfrac{5}{1-x} < 3$

IV Using the definition of the derivative, determine $f'(1)$ where $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ \sqrt{x} - 1 & \text{if } x \geq 1 \end{cases}$ (10 pts)

V Given $f(x) = |2x + 3| - |4 - x|$. (15 pts tot)

a) Rewrite $f(x)$ as a piece-wise function.

b) Sketch the graph of $f(x)$.

c) Determine $f'(x)$

VI Find the following limits: (5 pts ea)

a) $\lim\limits_{x \to 0^-} \dfrac{2x^2 + 3x}{|x|}$ b) $\lim\limits_{x \to -3} \dfrac{x^3 + 27}{x + 3}$ c) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$

d) $\lim\limits_{x \to 0} \dfrac{x^2 + \frac{1}{x}}{x^2 - \frac{1}{x}}$ e) $\lim\limits_{x \to 0} (1 - 3x)^{\frac{2}{x}}$ f) $\lim\limits_{x \to \infty} \sqrt{x^2 + x} - \sqrt{x^2 + 4}$

g) $\lim\limits_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4 - x & \text{if } x < 1 \\ \sqrt{6} & \text{if } x = 1 \\ 2 + x^2 & \text{if } x > 1 \end{cases}$

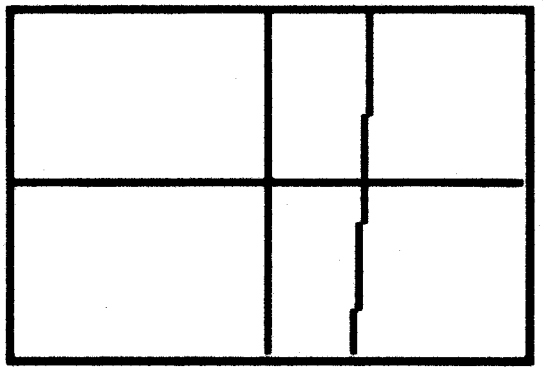
Window on the World

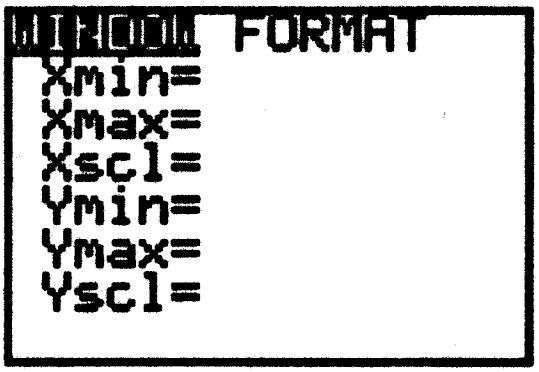
Mr. Mumaugh

Given $f(x) = x^3 - 2x^2 + x - 30$

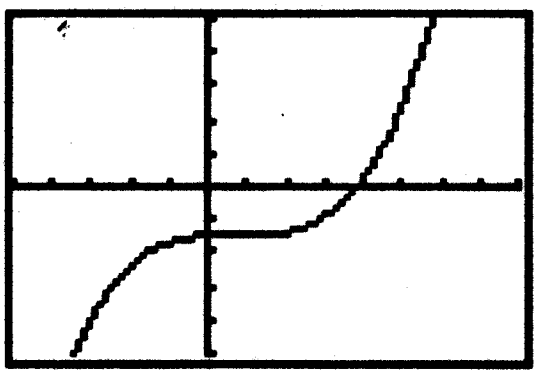
I Determine the "window" settings which gives the supplied picture. Your picture must be exact including the "ticks" on the axes. (hint: YES, these are all of the *same* function as stated above!

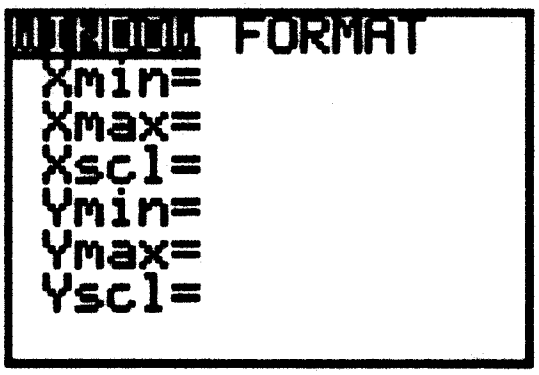
a)



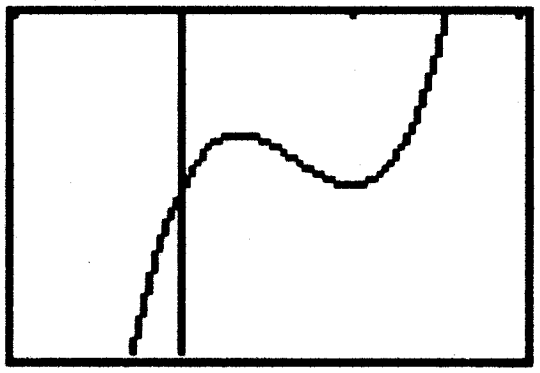


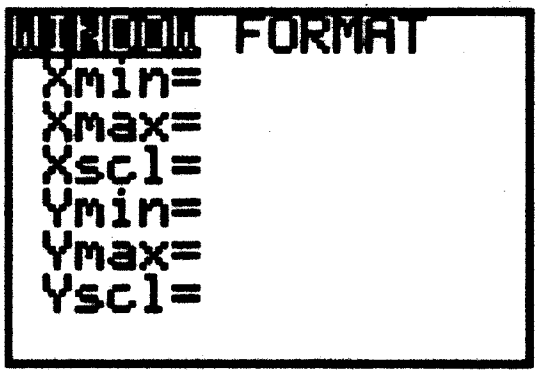
b)





c)





II What are the coordinates of the axis intercepts of the function?

III What are the coordinates of the relative max and the relative min shown in the graph of I c?

IV Why was this particular problem assigned? (What did you learn from it?) Be clear.

The world famous Canadian big league power hitter, Jasper leDog stepped up to the plate. He popped up the ball. He made contact with it 3 feet above the ground. The ball soared almost out of sight!

- What is the height of the ball after 2 seconds?
- When is the ball 80 feet above the ground?
- If the catcher misses the ball, when does it hit the ground.
- If the six-foot tall catcher catches the ball one foot above his head, when does it land in his mitt?
- How high is the ball at its highest point? When did it get there?

We will use the following function of height in terms of time: $h(t) = -16t^2 + 98t + 3$. The $-16t^2$ represents the effect of gravity on the ball. The $+98t$ represents the vertical velocity of the ball. That is, how hard the ball was hit straight up. Finally the $+3$ represents that the ball was hit by the bat 3 feet above the ground.

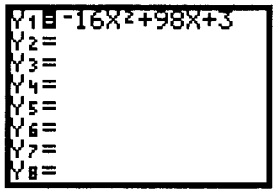
Turn on your calculator by pressing the ON button in the lower left hand corner.

We need to key in our function so press Y= in the upper left corner. The cursor is blinking on the Y_1 line. On your calculator there is a "subtract" key and a different key called "minus." The "subtract" key is in the right hand column between the "times" and "add" keys. The "minus" key is on the bottom row.

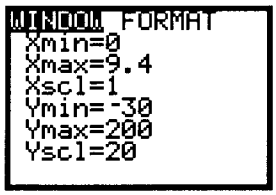
Press "minus"
Press 16
Press x,t,θ (third row, second column)

Press x^2 (row 6, column 1)
– – – – – – OR – – – – – –
Press ^ (above the ÷ key)
Press 2

Press "plus" (above the ENTER key)
Press 98
Press x,t,θ
Press "plus"
Press 3
Your screen should look like this:



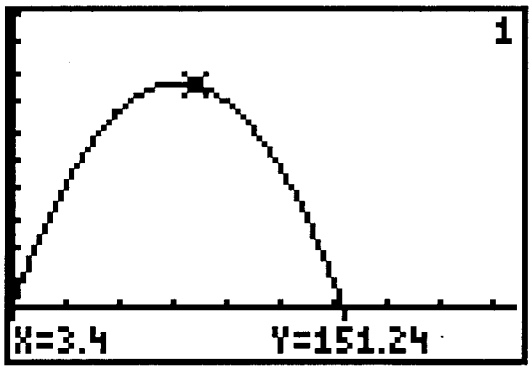
Press WINDOW
Press Down Arrow (or ENTER)
Press zero
Press ENTER
Press 9.4 etc. to get :



The screen width is 94 pixels so we like some decimal multiple of 94. The screen height is 62 pixels but we will not need to make adjustments for that at this time.

The first question was about the height of the ball after 2 seconds. We have 4 different methods to find out.

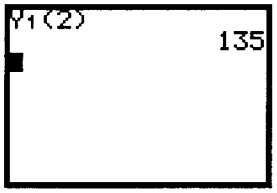
Method 1. Graphically: Press TRACE



Using the left arrow, move the cursor left until the x–value is 2. The y–value is the height desired.

Method 2. Using function notation in the HOME SCREEN.

Press 2nd QUIT (next to the blue 2nd button) This takes you to the Home Screen.
Press 2nd Y-VARS This lists the different kinds of functions available. The first item is the standard function and is highlighted so press ENTER.
This lists all the Y functions in the calculator. Since we only have one function, and it is already highlighted ...
Press ENTER This places the function Y_1 on the Home Screen.
Press (6th row, 3rd column
Press 2) ENTER

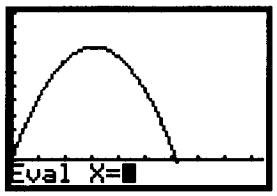


The function value is 135.

Method 3. Calculate the value using the CALC menu.

Press 2nd CALC (1st row 4th column)

Press ENTER (This selects option 1 which is value.

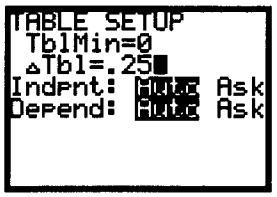


Press 2 ENTER

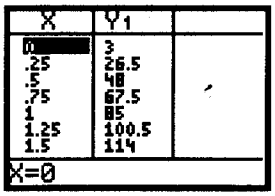
Method 4. Use a table of values.

Press 2nd TblSet row 1 column 2

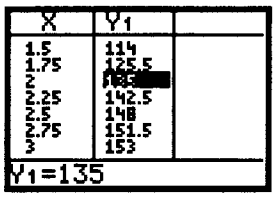
Use the ENTER and number and arrow keys to make your screen look like this:



Press 2nd TABLE Row 1 Column 5



Press the arrow keys until you see the desired values.



The next question was when was the ball 80 above the ground. There 2 ways to compute this. First we will define a second function, $f(t) = 80$.

Press Y=

Press ENTER or use the down arrow to get to the second function.

Press 80 Notice that the equal sign "inverted" when you put in the 80. When the equal sign is inverted, the function is active. To turn off or turn on a function, place the cursor on the = sign using the arrow keys and then press ENTER.

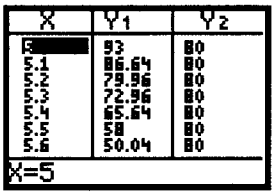
Press 2nd QUIT to get out of the function definition and to return to the Home Screen.

Method 1. We will zoom in using our table options.

Press 2nd TblSet and make TblMin 4 and ΔTbl= .1.

Press 2nd TABLE.

Press and hold the down arrow key. Watch the values of Y_1 and Y_2. When are they the same or very nearly the same?



5.2?

Press 2nd TblSet and make ΔTbl = .01

Press 2nd TABLE.

Use the arrow keys again to get Y_1 and Y_2 about equal.

Press 2nd TblSet and make ΔTbl = .001

Press 2nd TABLE.

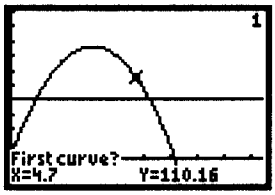
Use the arrow keys again to get Y_1 and Y_2 about equal.

Continue until you have an answer accurate to 4 digits.

Method 2. Using the Calculate Menu:

Press 2nd CALC

Move the cursor down to "intersect" and press ENTER or just press 5.



Press ENTER two times.

These identify the two functions of interest since it is possible for us to have many more than 2 functions on our screen.

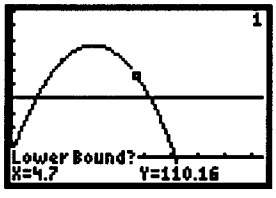
Move the cursor near the intersection point you want the calculator locate. Press ENTER.

The third question concerned when the ball would hit the ground. Three methods are available.

Method 1. Zoom in using the table as we did in method 1 of question 2 looking for the Y_1 value to be zero.

Method 2 Calculate the root of Y_1 since we are interested when the function is zero.

Press 2nd CALC. Press 2.



Press Enter to select that x value as lower bound.

Thenu se the right arrow to move the cursor so that it is *below* the x–axis. Press ENTER. Move the cursor to the x–axis to select it as your "guess" and press ENTER. (x = 6.1554608)

Method 3. We will use a programming instruction to do the job.

Press 2nd QUIT to get to the Home Screen.

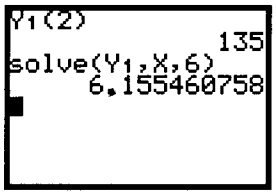
Press MATH row 4 column 1

Press the down arrow to see all the options. Select the option solve(by highlighting it and then pressing ENTER.

Locate your supplement commands for your calculator and find solve(.

Solve(*expression,var,guess*)

NOTE the comma is above the 7 button, the variable X comes from the 3rd row 2nd column and be sure to close your parenthesis and then press ENTER. We want Y_1. Start by pressing 2nd Y–VARS. Finish up your line. :



The fourth question about the catcher who does catch the ball you can do.

The fifth question was how high did the ball get and when did it do it?

Method 1 Use TblSet and TABLE and the method of zooming in on the largest value of Y_1.

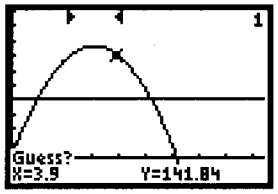
Before we begin this next method, I would like to "save" the current view so I can get back to it easily.

Press ZOOM. Press right arrow to highlight MEMORY.

Press 2. This stores the current window configuration for our later recall using option 3 from the list of choices ZoomRcl.

Method 2 Use the CALC function maximum.

After first moving the cursor on the left of the apparent maximum value location, pressing ENTER and moving the cursor to the right of the max location and pressing ENTER again:



Move the cursor to the apparent max and press ENTER a third time.

Method 3 We will look at the graph of the function and zoom in so we can see the exact spot.

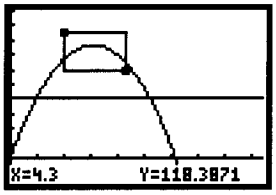
Press TRACE

Move the cursor so it is on the obvious maximum spot. Since we are not close enough, it is hard to tell what that height is.

Press ZOOM. Notice that the first line is highlighted. You can choose an option by pressing ENTER when the option is highlighted or by just pressing the number on the left of the option you want.

We want ZBOX. (Zoom Box). The cursor turns into a + sign. Move the + sign left of the high point a little and then up a little and press ENTER.

Move the cursor to the right and below of the high point:



Press ENTER the second time and the calculator will make the necessary WINDOW adjustments so that the box you made will fill the viewing screen.

Repeat the ZBox routine a couple of times. You should get close enough that the "curve" will look like a horizontal line. You should be at that maximum value.

Just for fun ...

Press WINDOW. look at the values.

Press ZOOM, select MEMORY and Press 3. We have restored our window.

Press WINDOW. look at the values now.

Press MODE, select SPLIT and then press GRAPH.

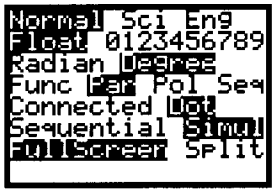
To see different possibilities for the bottom of the screen, press the following keys one at a time:

Y=, WINDOW, TABLE, 2nd QUIT, TRACE

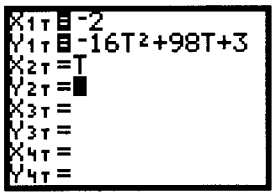
Motion Problems Using Parametric Equations

The graphing calculator can represent motion as parametric equations and show the position of an object in its flight path. Motion is assumed to be in a plane and air resistance is ignored.

Press MODE:



Press Y=:

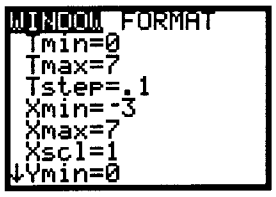


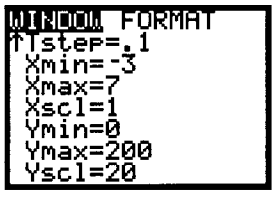
We want Y_{1T} where the cursor is now sitting.

Press 2nd Y-VARS, then 2 to select parametric and then 2 to select Y_{1T}.

The Y_{1T} is placed properly and the pair of equations becomes active (= signs are inverted).

Press WINDOW:





Now you see both the path of the ball (on the right) but also the height vs. time graph (on the left).

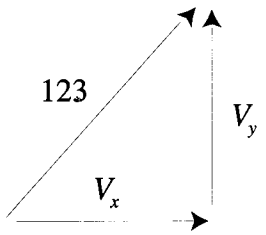
The "Green Monster," the left-field wall in Fenway Park in Boston, is 37 feet high and extends 210 feet towards center field, where its distance from home plate is 379. A baseball is hit by Jake leSnake (another famous canadian baseball player) when it is 3 feet from the ground with an initial velocity of 123 feet per second at an angle of elevation of 32 degrees.

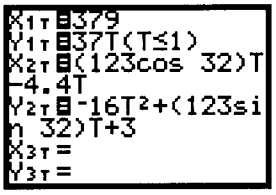
A wind of 3 mph (4.4 ft/sec) is blowing towards the batter. The ball is on a path that will take to the deepest part of the Green Monster, 379 feet away. Is it a home run? What is the smallest angle that would produce a home run? What angle would produce a ball which could be caught?

The initial force is resolved into the horizontal (x) and vertical (y) components using trigonometry:

$V_x = 123 \cos 32$ and $V_y = 123 \sin 32$

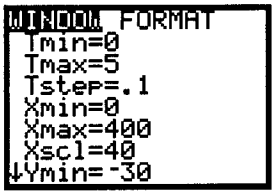
where:

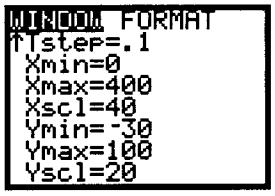




The line Y_{1T} means the wall is 379 from home plate.

You make the ≤ symbol using 2nd TEST (row 4) The wall is 37 feet tall so we want the calculator to plot points only up to 37 feet high. When T is larger than 1, 37T would be larger than 37 and the expression (T≤1) would be false. This is interrupted by the calculator as "equal to zero." That zero times the 37T still equals zero so the coordinate (379,0) gets turned on over and over. Until T = 1, (T≤1) is true and this is interpreted as a factor equal to plus one.





Press GRAPH to see the path of the ball.

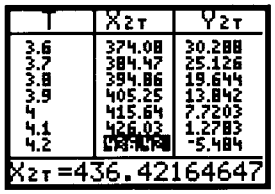
Was it a home run?

Next using TblSet set TblMin = 3 and ΔTbl = .1.

Press TABLE.

Press the right arrow 3 times to scroll the values of X_{2T} into view.

Scroll down to get:



T	X2T	Y2T
3.6	374.08	30.288
3.7	384.47	25.126
3.8	394.86	19.644
3.9	405.25	13.842
4	415.64	7.7203
4.1	426.03	1.2783
4.2	[illegible]	-5.484

X2T=436.42164647

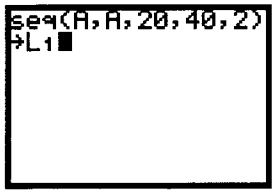
Note that 3.6 is the number in the upper left corner. This number will be moved into the TabMin spot. Press 2nd TblSet and make ΔTbl = .01. Press TABLE.

Repeat the process to close in on the X values near 379.

There is another operation the calculator handles which is unusal and it has to do with lists of values.

Lets start on the Home Screen. Press 2nd Quit.

Press 2nd List row 3 column 3 Press 5 to get Seq(



To get the A, press ALPHA. Press A row 4 col 1.

To get the comma, press , above the 7.

The short arrow means "store" and the STO key is in row 9.

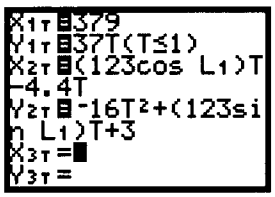
The L_1 means list # one and you get it by pressing 2nd 1.

When your screen looks like mine, press ENTER.

The meaning of the line is that we are first placing the value 20 in the variable A and placing that value in L_1 spot #1. Next we are adding 2 to A placing that value in L_1 spot #2 and so on until we get to 40.

To see the list itself and edit it, press STAT row 3 col 3 {NOTE you do not press 2nd STAT. I agree it seems logical to display a list by pressing LIST but the calculator treats lists as statistical data.} then press ENTER. You could edit any of the values in you list if you wanted to.

Next we need to **edit** our functions. Press Y=.



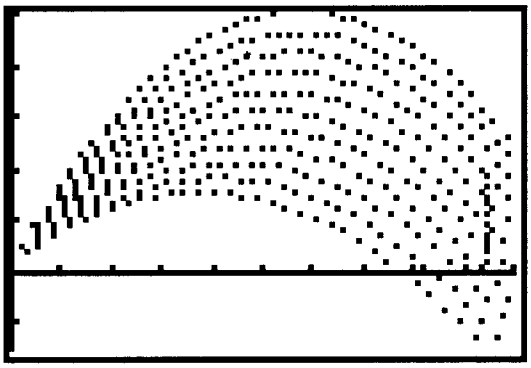
Notice the change in line X_{2T}.

I moved the cursor on top of the 3 in the 32 and pressed DEL row 2 col 3 two times, then I pressed 2nd INS row 2 col 3 also to put the calculator into insert mode. Next I pressed 2nd L_1.

Do the same for the next line.

What we have done is told the calculator to substitute the values from List #1 into our functions.

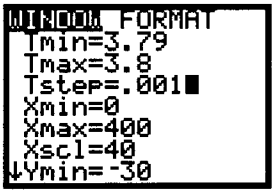
Press GRAPH.



Find the speed with which the ball in the original problem hits the wall and the angle it makes with the wall.

Change the L_1 back to 32 in X_{2T} and Y_{2T}.

Change WINDOW to T = [3.79, 3.8, .001] to get:



Use CALC dy/dt, dx/dt, and dy/dx.

Select dy/dt from CALC, trace to T = 3.794, the time that the ball hits the wall, and press ENTER to calculate.

Store dy/dt (Ans) in V by pressing 2nd ANS (row 10 col4) then STO (row 9 col 1) ALPHA V.

Similarly, store dy/dt in H.

Evaluate $\sqrt{\left(H^2 + V^2\right)}$. This is the speed of the ball when it hits the wall.

On the Home Screen enter $\tan^{-1}(V/H)$ (or find dy/dt and store this value) to find the angle the ball makes with the horizontal, then find the angle with the wall.

Try the above problems using MATH nDeriv instead of the CALC menu.